JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICAL & ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR THE BACHELORS DEGREE

2ND YEAR 1ST SEMESTER 2013/2014 ACADEMIC YEAR

CENTRE: MAIN

COURSE CODE: SMA 201

COURSE TITLE: LINEAR ALGEBRA II

EXAM VENUE: AH **STREAM: (BSc. Actuarial, Bed,B Sc)**

DATE: 16/4/2014 **EXAM SESSION: 9.00 – 11.00 AM**

TIME: 2 HOURS

Instructions:

1. **Answer question 1 (Compulsory)and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question1 [30marks] Compulsory

(a) Let $P_{n\times n}$ be a real square matrix.

(i)Define what is meant by $P_{n\times n}$ is orthogonal.

(ii) Let $P = \begin{pmatrix} 3 & -4 & 0 \\ 0 & 0 & 9 \\ 4 & 3 & 0 \end{pmatrix}$ be a real square matrix.

Prove that P is orthogonal with respect to the standard inner product of R^3 hence find P^{-1}, and $\hat{P}$ the orthonormalized form of P . [8 marks]

(b) Given the mapping $L: R^2 \to R^2$ with $L\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x-y \\ x+y \end{bmatrix}$ is a linear operator on R^2.

(i) Determine ker (L) (ii) Find A_L ,the matrix of L

(iii) Describe the rule for L^{-1} which is the inverse of L [8 marks]

(c) Without using direct computation , show that $\begin{pmatrix} -17 \\ -34 \\ 34 \end{pmatrix}$, $\begin{pmatrix} 10 \\ 10 \\ 0 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}$ are eigenvectors of

the matrix $A = \begin{pmatrix} 1 & -4 & -4 \\ 8 & -11 & -8 \\ -8 & 8 & 5 \end{pmatrix}$. Give the associated eigenvalues a_1, a_2, a_3 of this matrix.

Verify that $trace(A) = a_1 + a_2 + a_3$ [9 marks]

(d) Show that matrix $A = \begin{pmatrix} 1 & -4 \\ -9 & 1 \end{pmatrix}$ is diagonalizable but do not diagonalize A. [5 marks]

Question2 [20marks]

Let $A = \begin{bmatrix} 0 & 2 & -1 \\ 2 & 3 & -2 \\ -1 & -2 & 0 \end{bmatrix}$ define a matrix of linear operator T on R^3

(a) Find the characteristic polynomial of A. (5 marks)

(b) Without doing an eigenvalue–eigenvector computation, show that the vectors $u = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ are eigenvectors of A. [5 marks]

(c) Determine w the remaining eigenvector of A. (5 marks)

(d) Diagonalize matrix A. [5 marks]

Question3 [20marks]

(a) Let f be the form on $V \times V$ such that V is a real vector space. Define $A = \left(a_{ij}\right)$, the matrix of f w.r.t an ordered basis $\quad = \{\ _{1},\ \ _{2},,,\ \ _{n}\}$ by $A = \left(a_{ij}\right) = f\left(\ _{i},\ \ _{j}\right) i, j = 1, 2,,, n$.

Suppose f is a form on R^2 defined by

$f\left(\left(x_1, x_2\right), \left(y_1, y_2\right)\right) = x_1 y_1 + 4x_2 y_2 + 2x_1 y_2 + 2x_2 y_1$.

Find the matrix of f in each of the bases

(i) $\{[1,-1],[1,1]\}$ **(ii)** $\{[1,0],[0,1]\}$ ++++ [12 marks]

(b) Prove that the set 4by4 matrices $\left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}$ is linearly independent.

[8 marks]

Question4 [20 marks]

Consider the vector space of R^4 with the inner product $\langle\ ,\ \rangle$**:**

$\left\langle \underline{x}, \underline{y} \right\rangle = \frac{1}{2} x_1 y_1 + \frac{1}{2} x_2 y_2 + x_3 y_3 + x_4 y_4$**;** $\underline{x} = \left[x_1, x_2, x_3, x_4\right]$, $\underline{y} = \left[y_1, y_2, y_3, y_3\right]$, $x, y \in R^4$

(a)Apply the Gram-Schmidt process to the set of linearly independent vectors

$\{v_1 = [1,1,-1,-1], v_2 = [1,1,1,1], v_3 = [-1,-1,-1,1], v_4 = [1,0,0,1]\}$

to obtain orthogonal basis $\{w_1, w_2, w_3, w_4\}$**.** [16 marks]

(b) Obtain an orthonormal basis $\{u_1, u_2, u_3, u_4\}$ f or R^4. [4 marks]

Question5 [20 marks]

Let W be the space of all 3×3 matrices A over R which are skew- symmetric $i.e., A^t = -A$.

We equip W with the inner product $\left[A * B\right] = \frac{1}{2} tr\left[AB^t\right]$. Let V be the vector space R^3 with the standard inner product. If T be the mapping from V into W defined by

$$T\left(x, y, z\right) = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \quad i.e.\ T : V \to W$$

(a)Show that $T\left[u + kv\right] = Tu + kTv \quad for \quad u, v \in R^3$ [4 marks]

(b) Prove that T preserves the inner products V onto W [4 marks]