JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION SCIENCE/BACHELOR OF SCIENCE (ACTUARIAL SCIENCE WITH IT)

3RD YEAR 2ND SEMESTER 2016/2017 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SMA 303

COURSE TITLE: COMPLEX ANALYSIS

EXAM VENUE: AH | **STREAM: BED SCIENCE /SNE**

DATE: 20/04/17 | **EXAM SESSION:2.00 – 4.00 PM**

TIME: 2.00 HOURS

QUESTION ONE (COMPULSORY) – 30 MARKS

a) Define each of the following terms as used in complex analysis
   i) Argument
   ii) Principal argument
   iii) Limits of a complex function
   iv) Holomorphic functions (8 marks)

b) Find the image of a line $y = 2$ under the complex mapping $w = z^2$ for $w, z \in \mathbb{C}$, hence sketch the line and its image under the mapping (4 marks)

c) Express $-1-i$ in exponential form using the principal argument. (2 marks)

d) Determine the points of singularity for the function $f(z) = \dfrac{4z}{z^2 - 2z + 2}$ (4 marks)

e) Describe all the transformations represented by a complex mapping $f(z) = \sqrt{2}iz - 4 + 3i$ (4 marks)

f) Evaluate the line integral $I = \oint_c (x^3 dx + ydy)$ where C comprises the triangle $O(0,1)$, $A(1,2)$ and $C(0,0)$ (4 marks)

g) Compute the n[th] root for the $(1+\sqrt{3}i)^{\frac{1}{3}}$, hence sketch an appropriate circle indicating the roots w_0, w_1, . and w_2 (4 marks)

QUESTION TWO (20 MARKS)

a) Find the derivative of $\dfrac{iz^3 - 2z}{3z}$ (3 marks)

b) Evaluate $\left(\dfrac{\sqrt{3}+i}{i+1}\right)^4$, giving all your answers in polar form. (6 marks)

c) Compute the principal value of the complex logarithm $\ln z$ for $z = -i$ (4 marks)

d) Prove that if a complex function $f(z) = u(x,y) + iv(x,y)$ is analytic at any point z, and in the domain D, then the Laplace's equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$, can be verified. (7 marks)

QUESTION THREE (20 MARKS)

a) Evaluate the integral $\oint_c \dfrac{z}{z^2+9} dz$, where C is the circle $|z - 2i| = 4$ using the Cauchy's integral formular. (5 marks)

b) Use the definition of the derivative of a complex function to determine the derivative of $f(z) = z^2 - 1$ in the region where the derivative exists.
(5 marks)

c) Solve for w, given the complex function $e^w = \sqrt{2}i$ for $w, \in \mathbb{C}$. (5 marks)

d) State De-Moivre's theorem hence use it to evaluate $(\sqrt{6} - 3\sqrt{2}i)^6$, giving your answer in the form $a + bi$, $a, b \in \mathbb{R}$ (5 marks)

QUESTION FOUR (20 MARKS)

a) Find the value of i^i (4 marks)

b) Show that the n^th^ of unity are given by $(1)^{\frac{1}{n}} = \cos\frac{2k\pi}{n} - i\sin\frac{2k\pi}{n}, k = 0,1,2,.....(n-1)$
(6 marks)

c) Solve the compex quadratic equation $z^2 - (1+9i)z - 20 + 5i = 0$
(4 marks)

d) State the Cauchy's integral formular for derivatives hence evaluate

$$\oint \frac{z^3 + 3}{z(z-i)^2}$$

(6 marks)

QUESTION FIVE (20 MARKS)

a) Given the complex function $f(z) = u(x, y) + iv(x, y)$, verify that the function $u(x, y) = x^2 + 4x - y^2 + 2y$ is harmonic hence find $v(x, y)$ the harmonic conjugate u,Hence find the corresponding analytic function $f(z) = u + iv$. (6 marks)

b) Evaluate $\oint \frac{1}{z} dz$, where $C$is the circle $x = \cos t, x = \sin t$ for $0 \le t \le 2\pi$
(4 marks)

c) Show that the function $f(z) = 3x^2y^2 - 6ix^2y^2$ is not analytic at any point but differentiable along the coordinate axes.
(5 marks)

d) Use L'Hopital's rule to compute

$$\lim_{z \to 1+i} \frac{z^5 + 4z}{z^2 - 2z + 2}$$

(5 marks)