JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

2ND YEAR 2ND SEMESTER 2016 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SMA 208

COURSE TITLE: INTRODUCTION TO ANALYSIS

EXAM VENUE: LAB 1

STREAM: (BSc. Actuarial)

DATE: 05/09/16

EXAM SESSION: 9.00 – 11.00 AM

TIME: 2.00 HOURS

QUESTION ONE [30 Marks]

a) Give the formal definition of a proper subset

(2mks)

b) Let A and B be two non-empty sets then prove that

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

(5mks)

c) If $A = \{2, 3, 9, x, y\}$ and $B = \{1, 3, 5, 9, y\}$, determine $B \backslash A$

(2mks)

d) Let A, B and C be non empty sets, show that if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

(3mks)

e) Let X and Y be two open sets. Show that their intersection is open.

(6mks)

f) Given that $X = \{2, 3, x\}$, determine its cardinality and the power set of X.

(4mks)

g) Prove that $(A \cup B)^c = A^c \cap B^c$.

(8mks)

QUESTION TWO [20 Marks]

(a) Given a point $x_0 \in X$ and a real number $r > 0$ define the following:

(i) open ball

(ii) Closed ball

(iii) Open Set

(iii) closed Set

(7mks)

(b) Given that Q is a set such that $Q = \{(x, y, z, w) | w < 2\}$ Show that it is open.

(6mks)

(c) Let K be a set. Show that K is closed iff K^c is open.

(7mks)

QUESTION THREE [20 Marks]

(a) Define the following

(i) an upper bound of a subset S of a partially ordered set $(P, \leq)$

(2mks)

(ii) infimum a subset S of a partially ordered set $(P, \leq)$

(3mks)

(b) if N is a set such that $N = \{\frac{1}{n} - 1 : n \in \mathbb{N}\}$. Find the
Upper bound
lower bound
infimum
supremum

(4mks)

(c) Let S be a bounded set, if the infimum does exist, show that it is unique.

(6mks)

(d) Find the

$$\lim_{n\to\infty} \frac{n^3 - 4n + 2}{n^3}$$

(5mks)

QUESTION FOUR [20 Marks]

(a) Define a rational number.

(2mks)

(b) Show that there is no rational number whose square is 8.

(6mks)

(c) By showing all the axioms of a field, determine whether the set of rational numbers is a field.

(12mks)

QUESTION FIVE [20 Marks]

(a) Define a continuous $f : X \to Y$ on $\mathbb{R}$.

(2mks)

(b) Let $f(x) = 7x - 5$. Prove that f is uniformly continuous on $\mathbb{R}$.

(5mks)

(c) Find the inverse function of

$$f(x) = \frac{4x - 2}{3}$$

(5mks)

(d) Let $f(x) = x^2 + 3x - 4$ and $g(x) = x + 2$. Show that $(f \circ g)(x) \neq (g \circ f)(x)$

(8mks)