JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF   SCIENCE ACTUARIAL

3RD YEAR 1ST SEMESTER 2016/2017 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SAC 301

COURSE TITLE: METHODS OF ACTUARIAL INVESTIGATIONS I

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

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Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

(a) Define the following terms as used in Financial Mathematics.

    i. Immunisation.

    ii. Macauly duration.

    iii. Continuous rate.

    iv. Certificate of deposit.

    v. Ex-dividend share. [5 marks]

(b) A ten-year bond with half yearly coupons of 6% pa has just been issued with a redemption yield of 9% pa effective. It is redeemable at par. What price would an investor paying 15% tax on income pay for the bond? Tax payments are due four months after each coupon is received. [5 marks]

(c) Calculate, using 10% pa interest, the convexity of the following assets, each of which has a discounted mean term of 11 years. Comment on your answers.

    i. Asset A is an 11-year zero coupon bond.

    ii. Asset B will provide a lump sum payment of 9,663 in 5 years time and a lump sum payment of 26,910 in 20 years time.

    iii. Asset C is a level annuity of 1 pa payable annually in arrears for 50 years. [9 marks]

(d) Consider a fixed interest security that pays coupons of 10% at the end of each year and is redeemable at par at the end of the third year. Calculate, using an interest rate of 8% p.a, the

    i. volatility of the cashflows. [2 marks]

    ii. discounted mean term of the cachflows. [2 marks]

    iii. convexity of the cashflows. [2 marks]

(e) A loan of nominal amount 100,000 is to be issued bearing coupons payable quarterly in arrear at a rate of 7% per annum. Capital is to be redeemed at 108 per 100 nominal on a coupon date between 15 and 20 years inclusive after the date of issue. The date of redemption is at the option of the borrower. An investor who is liable to income tax at 25% and capital gains tax at 40% wishes to purchase the entire loan at the date of issue.

    i. Determine the price which the investor should pay to ensure a net effective yield of at least 5% per annum. [4 marks]

    ii. Explain the significance of the redemption date being at the option of the borrower in relation to your calculation in part (i). [1 mark]

QUESTION TWO

(a) You are given the following information about the term structure of interest rates:
The one year spot rate is 4.1% p.a.
The two year spot rate is 4.0% p.a.

The four year spot rate is 4.4% p.a.
The five year spot rate is 4.4% p.a.

i. Calculate the theoretical price of a three year zero coupon bond that is issued at time 2 and is redeemable at par. [3 marks]

ii. A one year zero coupon bond is issued at time three and has a theoretical price of 95.95 per 100 nominal. Given that the bond is redeemable at par, calculate
   A. the 3-year spot rate
   B. the one year forward rate starting at time two. [3 marks]

iii. Calculate the gross redemption yield on a bond that pays annual coupons of 6% p.a at times 1, 2, 3, 4 and 5 and is redeemable at 110% in 5 year's time. [4 marks]

(b) The current annual term structure of interest rates is: (6%; 6%; 6%; 6%; 7%) Calculate the gross redemption yield of a five-year fixed-interest security redeemable at par if the annual coupon is

i. 2%. [2 marks]

ii. 4%. [2 marks]

(c) Evaluate the discounted mean term of a bond redeemable at par in 10 years time with annual coupons of 8% at interest rates of 5%, 10% and 15%. Hence sketch a graph of the discounted mean term as a function of the interest rate over the range 5% to 15%. [6 marks]

QUESTION THREE

(a) In a particular bond market, the two-year par yield at time $t = 0$ is 4.15% and the issue price at time $t = 0$ of a two-year fixed interest stock, paying coupons of 8% annually in arrears and redeemed at 98, is 105.40 per 100 nominal. Calculate:

i. the one-year spot rate

ii. the two-year spot rate. [6 marks]

(b) An insurance company has liabilities of 6 million due in exactly 8 years time and a further 11 million due in exactly 15 years time. The assets held by the insurance company consist of: a 5-year zero-coupon bond of nominal amount 5.5088 million; and a 20-year zero-coupon bond of nominal amount 13.7969 million. The current rate of interest is 8% per annum effective at all durations.

i. Show that the first two conditions of Redingtons theory for immunisation against small changes in the rate of interest are satisfied. [2 marks]

ii. Explain, without doing any further calculations, whether the insurance company will be immunised against small changes in the rate of interest. [2 marks]

(c) An investor has earned a money rate of return from a portfolio of bonds in a particular country of 1% per annum effective over a period of ten years. The country has experienced deflation (negative inflation) of 2% per annum effective during the period. Calculate the real rate of return per annum over the ten years. [2 marks]

(d) The one-year forward rates for transactions beginning at times $t = 0, 1, 2$ are f_t where

$$f_0 = 0.06, \qquad\qquad , f_1 = 0.065 \qquad\qquad f_2 = 0.07$$

Calculate the par yield for a 3-year bond. [3 marks]

(e) Describe the main features of:

    i. debenture stocks.

    ii. unsecured loan stocks. [5 marks]

QUESTION FOUR

(a) An economist's model of interest rate indicates that the n-yr spot rate of interest is $0.1(1+e^{-0.1n})^{-1}$ According to this model, what is the price of a 10 year zero coupon bond redeemable at par?. [3 marks]

(b) An investor purchases a bond 3 months after issue. The bond will be redeemed at par ten years after issue and pays coupons of 6% per annum annually in arrears. The investor pays tax of 25% on both income and capital gains (with no relief for indexation). Calculate the purchase price of the bond per 100 nominal to provide the investor with a rate of return of 8% per annum effective. [6 marks]

(c) For the last 10 years a man has paid 50 at the start of each month into a savings account that has achieved a real rate of interest of 3% per annum over this period. If inflation has been at a constant rate of 5% per annum, calculate the balance of the man's account today. [4 marks]

(d) A loan of nominal amount 100,000 is to be issued bearing coupons payable quarterly in arrear at a rate of 7% per annum. Capital is to be redeemed at 108 per 100 nominal on a coupon date between 15 and 20 years inclusive after the date of issue. The date of redemption is at the option of the borrower. An investor who is liable to income tax at 25% and capital gains tax at 40% wishes to purchase the entire loan at the date of issue.

    i. Determine the price which the investor should pay to ensure a net effective yield of at least 5% per annum. [5 marks]

    ii. Explain the significance of the redemption date being at the option of the borrower in relation to your calculation in part (i) [2 marks]

QUESTION FIVE

(a) The n-year spot rate of interest s_n is given by the formula

$$s_n = 0.05 - \frac{n}{500}, \qquad\qquad 1, 2, 3$$

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    i. Calculate the implied 1-year forward rates applicable at times $t = 1$ and $t = 2$ [3 marks]

ii. An investor purchases a three year coupon bond that provides coupons of 6% p.a payable annually in arrears and is redeemable at par. Show that the fair price of this bond is 104.36 per 100 nominal. [1 mark]

iii. Calculate the investors gross redemption yield. [3 marks]

iv. Calculate the par yield of the bond. [3 marks]

(b) Explain how expectations theory can be modified by both liquidity preference and market segmentation theories. [6 marks]

(c) A stock with a term of 9.5 years has a coupon of 5% *pa* payable half yearly in arrears and is redeemable at 105%. An investor who is not subject to tax purchases the stock at 85 per 100 nominal immediately after the coupon payment. Calculate the yield obtained by the investor. [4 marks]