# JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY <br> SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE <br> UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE <br> ACTUARIAL <br> $3^{\text {RD }}$ YEAR $1^{\text {ST }}$ SEMESTER 2016/2017 ACADEMIC YEAR <br> REGULAR (MAIN) 

COURSE CODE: SAC 303
COURSE TITLE: ACTUARIAL MATHEMATICS II
EXAM VENUE:
STREAM: (BSc. Actuarial)
DATE:
EXAM SESSION:
TIME: 2.00 HOURS

## Instructions:

1. Answer question 1 (Compulsory) and ANY other 2 questions
2. Candidates are advised not to write on the question paper.
3. Candidates must hand in their answer booklets to the invigilator while in the examination room.

## QUESTION ONE

(a) Define the following terms as used in actuarial mathematics
i. Multiple state model.
ii. Multiple decrement model.
iii. Contingent probability.
[3 marks]
(b) A population is subject to two modes of treatment $\alpha$ and $\beta$. Both decrements are uniformly distributed over each year of age in the single decrement table.
i. Show that $(a q)_{x}^{\alpha}=q_{x}^{\alpha}\left(1-\frac{1}{2} q_{x}^{\beta}\right)$ for integer ages of $x$.
[5 marks]
ii. A population is subject to two modes of decrement, $\alpha$ and $\beta$, between ages ( $x$ and $(x+1)$. In the single decrement tables

$$
{ }_{t} p_{x}^{\alpha}=\left(\frac{x}{x+t}\right)^{2}, \quad{ }_{t} p_{x}^{\beta}=\left(\frac{x}{x+t}\right)^{3}, 0 \leq t \leq 1
$$

Write down an integral expression for $(a q)_{x}^{\alpha}$. Hence or otherwise obtain an expression for this probability in terms of $x$ only.
Calculate

$$
(a p)_{60} \quad{ }_{2 \mid}(a q)_{60}^{\alpha}
$$

[6 marks]
(c) Two lives are both aged 45 exact. Given that $\mu_{x}=0.05$ for all $x$ for both lives and the rate of interest is $4 \%$ p.a, calculate
i. The probability of both lives surviving to age 65 exact.
[2 marks]
ii. The present value of annuity of 1000 p.a increasing by $3 \%$ each year payable annually in advance so long as both lives survive.
[4 marks]
iii. The present value of a 20-year term assurance with a benefit of 100000 payable immediately on the second death.
[5 marks]
(d) A multiple decrement table is subject to two forces of decrement $\alpha$ and $\beta$. Under the assumption of a uniform distribution of the independent decrements over each year of age, $(a q)_{x}^{\alpha}=0.2$ and $(a q)_{x}^{\beta}=0.05$. Calculate $q_{x}^{\alpha}$ and $q_{x}^{\beta}$.
[2 marks]
(e) The staff of a company are subject to two modes of decrement, death and withdrawal from employment. Decrements due to death take place uniformly over the year of age in the associated single-decrement table: $50 \%$ of the decrements due to withdrawal occur uniformly over the year of age and the balance occurs at the end of the year of age, in the associated single-decrement table. You are given that the independent rate of mortality is 0.001 per year of age and the independent rate of withdrawal is 0.1 per year of age. Calculate the probability that a new employee aged exactly 20 will die as an employee at age 21 last birthday.
[3 marks]

## QUESTION TWO

(a) i. State the assumptions underlying the binomial mortality model.
[2 marks]
ii. A cat has nine lives, so the cat will not die until it has lost all the nine lives. The probability of a cat loosing a life is $20 \%$ per week. Assuming that the mortality of each life follows the binomial model, calculate the probability that a cat who has currently lost all its nine lives will die during the next ten years.
[4 marks]
(b) Explain the rationale behind the use of the Poisson distribution to model the number of deaths among a group of lives. Include in your explanation a discussion of why the Poisson Model is not always an exact model.
[4 marks]
(c) On 1 January of a particular year, there were 406 men and 418 women in the age- range 25 to 30 living in a small town. If the initial rate of mortality can be assumed to be constant in this age - range. 4.2 per 10000 for men and 3.3 per 10000 for women, calculate the probability that exactly two of these lives will die during that year.
[5 marks]
(d) The table below shows the independent rates of ill-health retirements, withdrawals and deaths for a pension scheme for ages 20 and 40. Calculate the dependent rates of decrement at these ages, assuming that each decrement is uniform over each year of age in its single decrement table.

| Age | Ill - health | Withdrawal | Death |
| :---: | :---: | :---: | :---: |
| 20 | - | 0.25 | 0.002 |
| 40 | 0.01 | 0.05 | 0.003 |

[5 marks]

## QUESTION THREE

(a) Using the PMA92C20 for both lives, calculate
i. $\mu_{65: 60}$
ii. ${ }_{5} p_{65: 60}$
iii. ${ }_{2} q_{65: 65}^{1}$
[6 marks]
(b) A male aged 52 exact and a female aged 50 exact take out a whole life insurance policy . The policy pays a sum assured of 100000 immediately on the first death. Premiums are payable for a period of five years, monthly in advance. Calculate the monthly premium payable assuming a PMA92C20 for male life and PFA92C20 for the female life and an interest rate of $4 \%$.
[7 marks]
(c) The decrement table extract below is based on the historical experience of a very large multinational company's workforce.

| Age $(x)$ | $(\mathrm{al})_{x}$ | $(\mathrm{ad})_{x}^{d}$ | $(\mathrm{ad})_{x}^{w}$ |
| :---: | :---: | :---: | :---: |
| 40 | 10000 | 25 | 120 |
| 41 | 9855 | 27 | 144 |
| 42 | 9684 |  |  |

Recent changes in working conditions have resulted in an estimate that the annual independent rate of withdrawal is now $78 \%$ of that previously used. Calculate a revised table assuming no changes to the independent death rates.
[7 marks]

## QUESTION FOUR

(a) Suppose that in a triple decrement model, you are given constant forces of decrement for $(x)$ as follows

$$
\begin{gathered}
\mu_{x+t}^{(1)}=b, \quad \mu_{x+t}^{(2)}=b, \quad \mu_{x+t}^{(3)}=2 b, \text { for } t>0 \\
{ }_{3} q_{x}^{(1)}=0.00884
\end{gathered}
$$

Find the length of time a life aged $(x)$ now is expected to remain in the decrement table. [7 marks]
(b) You are given that in a certain population
i. Mortality of males has a constant force of mortality $\mu=0.004$
ii. Mortality of females follows a De Moivre's law with $\omega=115$.

Calculate the probability that a male aged 65 today will outlive a female aged 65 today. [7 marks]
(c) You are given:
i. Male mortality follows De Moivres law with $\omega=90$.
ii. Female mortality also follows De Moivres law where at age 80, the force of mortality is half that of the male force of mortality.
For two independent lives, a male age 75 and a female age 80 , determine the expected time until the second death.
[6 marks]

## QUESTION FIVE

(a) A continuous two-life annuity pays
i. 100 while both (30) and (40) are alive,
ii. 70 while (30) is alive but (40) is dead, and
iii. 50 while (40) is alive but (30) is dead.

The actuarial present value of this annuity is 1180 . Continuous single life annuities paying 100 per year are available for (30) and (40) with actuarial present values of 1,200 and 1,000 , respectively. Calculate the actuarial present value of a two-life continuous annuity that pays 100 while at least one of them is alive.
[5 marks]
(b) Consider the following healthy sickness model;


Given the following transition intensities, $\mu^{01}=0.003, \mu^{02}=0.006, \mu^{03}=\mu^{02}$, calculate the probability that
i. You will stay alive within four years.
[2 marks]
ii. Within four years, you will die of cancer.
[2 marks]
iii. Given that you did within four years, what is the probability that your cause of death was cancer?
[4 marks]
iv. The probability that you eventually die of cancer.
[2 marks]
(c) Assume that for two independent lives (50) and (60), mortality is described by

$$
\mu_{z}=\frac{1}{100-z} \text { for } 0 \leq z<100
$$

Calculate ${ }^{\circ} 50: 60$ and interpret this value.
[5 marks]

