MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2014/2015

FOURTH YEAR SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

SUPPLEMENTARY

SMA 404: COMPLEX ANALYSIS II

Date: 14th November, 2015 Time: 8.30 - 10.30am

INSTRUCTIONS:

- Answer Question ONE and any other TWO Questions

QUESTION ONE (30 MARKS)

(a) If z_1 and z_2 are any two complex numbers, show that

(i) $|z_1 + z_2| \leq |z_1| + |z_2|$

(ii) $|z_1 - z_2| \geq |z_1| - |z_2|$ [6 Marks]

(b) Evaluate the integral

$$\int_C \frac{z}{(9-z^2)(z+i)}\, dz$$

where C is the circle $|z| = 2$ described in the positive sense. [5 Marks]

1. (c)] If $f(z)$ and $g(z)$ are analytic in a domain D and continuous on the boundary curve C, show that $f(z) = g(z)$ for all $z \in D$. [2 marks]

(d) Find the value of the integral

$$\int_0^{1+i} (x - y + ix^2)\, dz$$

along the real axis from $z = 0$ to $z = 1$ and then along a line parallel to the imaginary axis from $z = 1$ to $z = 1 + i$. [5 marks]

(e) Show that an analytic function which is not identically zero can have only isolated zeros. [5 marks]

(f) The function $f(z)$ has a double pole at $z = 0$ with residue 2, a simple pole at $z = 1$ with residue 2, is analytic at all other finite points of the plane and is bounded on $|z| \to \infty$. Also $f(2) = 5$ and $f(-1) = 2$. Find $f(z)$. [7 marks]

QUESTION TWO (20 MARKS)

Evaluate, using the calculus of residues:

(a) $\int_0^{2\pi} \frac{d\theta}{1+a^2-2\cos\theta}$ $(0 \leq a < 1)$ [10 Marks]

(b) $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^3}$ [10 Marks]

QUESTION THREE (20 MARKS)

(a) Let $f(z)$ be analytic in a simply connected domain D bounded by a rectifiable Jordan arc C and be continuous on C. Show that

$$f(z) = \frac{1}{2\pi i} \int_C \frac{f(w)}{w - z}\, dw \quad \text{for all } z \in D.$$

[10 marks]

(b) In part (a) above, show that the derivative function $f'(z)$ is analytic in D.

[10 Marks]

QUESTION FOUR (20 MARKS)

(a) If $f(z)$ is analytic in the doubly connected region D defined by

$$\rho < |z-a| < R.$$

Show that $f(z)$ can be expressed in a Laurentz series

$$f(z) = \sum_{n=-\infty}^{\infty} a_n(z-a_n)$$

where a_n's are constants. [10 Marks]

(b) Find the Taylor's and Laurent's series which represent the function

$$\frac{1}{z(z^2-3z+2)}$$

when

(i) $0 < |z| < 1$

(ii) when $1 < |z| < 2$

(iii) when $|z| > 2$.

[10 Marks]

QUESTION FIVE (20 MARKS)

(a) Show (using Liouville's theorem) that every polynomial of degree ≥ 1 has at least one zero. [10 Marks]

(b) Explain the norm: $z = a$ is an isolated removable singularity of a function $f(z)$. [3 marks]

(c) If $z = a$ is an isolated singularity of $f(z)$ and if $|f(z)|$ is bounded on some deleted neighbourhood of a, show that a is a removable singularity. [7 marks]