MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

THIRD YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION SCIENCE WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

SPH 313: CLASSICAL MECHANICS

Date: 3rd December, 2016 Time: 8.30 – 11.30 am

INSTRUCTIONS:

- Answer ALL questions in SECTION ONE and any TWO questions in SECTION TWO.

SPH 313: CLASSICAL MECHANICS

INSTRUCTION TO CANDIDATES

- ✓ *Attempt QUESTION ONE in SECTION 1 and ANY TWO QUESTIONS from SECTION 2.*

SECTION 1: COMPULSORY [30MRKS]

QUESTION ONE [30MRKS]

a) State the conservation theorem for the angular momentum of a particle. [1mrk]

b) Distinguish between holonomic and nonholonomic constraints. [2mrks]

c) What is meant by a configuration space in classical mechanics? [1mrk]

d) If a rod travels with a speed $v = 0.8c$ along its length, how much does it shrink? [2mrks]

e) In a science fiction movie, two spaceships are moving head-on towards each other with speeds $0.65c$ and $0.90c$ with respect to an observer on earth. What is the relative speed measured by the astronauts on earth ship? [2mrks]

f) Show that the shortest distance between two points in a plane is a straight line of the form $y = ax + b$, where a and b are the constants of integration. [3mrks]

g) State the D'Alembert's Principle. [1mrk]

h) State any two types of conservative forces. [1mrk]

i) What is twin paradox? [1mrk]

j) State the two postulates of special theory of relativity. [2mrks]

k) What is Poincaré group? [1mrk]

l) You are given the natural form of Lagrangian as $L = \frac{1}{2}m\left(\dot{x}^2 - \omega_o^2 x^2\right)$. Find the Hamiltonian. [2mrks]

m) If we have a function $f(y, \dot{y}, x)$ defined on a path $y = y(x)$ between two values x_1 and x_2, where $\dot{y}$ is the derivative of y with respect to x, state the Euler-Lagrange differential equation. [1mrk]

n) What is an inertial reference frame? [1mrk]

o) Let the relativistic momentum of a particle be $p = mv = m_o v\left(\sqrt{1 - \frac{v^2}{c^2}}\right)^{-1}$ and the Einstein energy formula $E = mc^2$. Show that $E^2 = p^2c^2 + m_o^2c^4$. [3mrks]

p) Define a virtual displacement. [1mrk]

q) Explain any two limitations of Newtonian Mechanics. [1mrk]

r) State any two advantages of a variational principle formulation. [1mrk]

s) If you are given the potential energy of a system as $V(r) = \frac{-k}{r} + \frac{h}{r^2}$, show that at equilibrium $(r = r_o)$ $k = \frac{2h}{r_o}$. [3mrks]

SECTION 2: ATTEMPT ANY TWO QUESTIONS [40MRKS]

QUESTION TWO [20MRKS]

a) If the angular momentum of a particle about point O is given as $\bar{L} = \bar{r} \times \bar{P}$, and the moment of force or torque about the same point is given as $\bar{N} = \bar{r} \times \bar{F}$, show that $\bar{N} = \dot{\bar{L}}$. [4mrks]

b) Consider a one-dimensional situation in which there is a force $F(x)$ that depends on one coordinate only and is therefore a conservative force. If a particle moves under this force, show that the total energy is conserved and is given by $T + V = T_o + V_o = E$ (constant). [9mrks]

c) You are provided with the kinetic energy and potential energy for a one-dimensional harmonic oscillator as $T = \frac{1}{2} m\dot{x}^2$, and $V = \frac{1}{2} kx^2$ respectively. Show that the standard harmonic oscillator equation is given by $\ddot{x} + \frac{k}{m} x = 0$, where $\omega = \sqrt{k/m}$ is the circular frequency of the oscillator. [7mrks]

QUESTION THREE [20MRKS]

a) Explain the two types of difficulties constraints introduce in the solution of mechanical problems. [4mrks]

b) Use the principle of virtual work to derive D'Alembert's Principle. [9mrks]

c) Show that the one-dimensional harmonic oscillator whose Hamiltonian equation $H = \frac{1}{2m}\left(p^2 + m^2\omega^2 x^2\right)$ is a constant motion that flows as an ellipse in a two-dimensional plane with x and p as coordinates. Further, by rescaling one of the coordinates, show that the system's constant motion forms circles. Sketch these circles for one-dimensional harmonic oscillator giving conditions for the direction of flow. [7mrks]

QUESTION FOUR [20MRKS]

a) If the Lagrangian of a system is given as $L = \frac{1}{2} m\dot{x}^2 - U(x)$, find the Hamiltonian. [3mrks]

b) The motion of one particle using Cartesian coordinates has its kinetic energy as $T = \frac{1}{2} m\left[\dot{x}^2 + \dot{y}^2\right]$.

i. By letting $x = r\cos\theta$, $y = r\sin\theta$, show that T can be transformed to polar coordinates as $T = \frac{1}{2} m\left(\dot{r}^2 + r^2\dot{\theta}^2\right)$. [4mrks]

ii. Use the transformed D'Alembert's Principle to find the equations of motion of (i) above. [6mrks]

c) Consider a particle of mass m moving in a two-dimensional central force field $\bar{F} = \lambda\bar{r}$, where $\bar{r} = x\bar{e}_x + y\bar{e}_y$ and λ is a positive constant. At an initial time $t = 0$, $\bar{r}_o = a\bar{e}_x + b\bar{e}_y$ where a and b are positive constants. The initial velocity is not known. However, the product of velocity components does not depend on time and equals to a non-zero constant all the time. Show that the particle moves in a hyperbola. [7mrks]

QUESTION FIVE [20MRKS]

a) State the Lorentz coordinate transformation which gives the correct relativistic formula for the transition between the inertial frames S and S'. [2mrks]

b) What is time dilation in relativistic mechanics? [1mrk]

c) Write down the relativistic mass formula. [1mrk]

d) Study the Atwood's machine in **Fig. 1** below and answer the questions that follow.

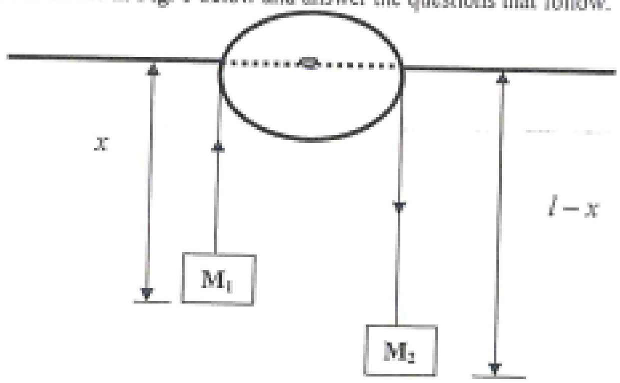

Fig. 1: Atwood's Machine

i. Is the system conservative or non-conservative? [1mrk]

ii. If the pulley is assumed frictionless and massless, state the types of constraints present. [2mrks]

iii. Write down both the potential and kinetic energies of the system. [2mrks]

iv. Set up the Lagrangian of the system. [2mrks]

v. Determine the equation of motion of the system. [2mrks]

e) A uniform coin of mass M and radius R stands vertically on the right end of a horizontal uniform plank of mass M and length L as shown in **Fig. 2**. The plank is pulled to the right with a constant force $\vec{F}$. Assume that the coin does not slip with respect to the plank. What are the accelerations of the plank and the coin? [7mrks]

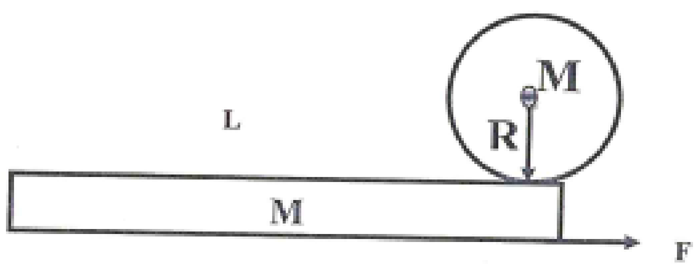

Fig. 2: A coin on a uniform plank