MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2015/2016

FIRST YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF MASTER OF SCIENCE IN PHYSICS

MAIN CAMPUS

SPH 802: CLASSICAL ELECTRODYNAMICS

Date: 16th December, 2015 Time: 9.00 - 12.00noon

INSTRUCTIONS:

- Answer any THREE Questions.

2015/2016 ACADEMIC YEAR SEPTEMBER-DECEMBER 2015 SESSION

SPH 802: CLASSICAL ELECTRODYNAMICS

Answer any **THREE** questions

QUESTION 1 (20 MARKS)

(a) Describe briefly the origin and geometrical form of an electromagnetic field.

(b) Given that an electromagnetic field characterized by a field potential four-vector $A = (\phi, A)$ is specified by a set of field equations $\frac{1}{c}\frac{\partial \phi}{\partial t} + \vec{\nabla}.\vec{A} = 0; \vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}; \vec{B} = \vec{\nabla} \times \vec{A}$ where the first equation is the Lorentz gauge condition, while $\vec{E}$ and $\vec{B}$ are the usual field intensities.

(i) Show that the full set of Maxwell's equations in a general medium containing electric charge and current density are derivable from the field equations under the wave propagation conditions $\frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \phi = \rho; \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} - \nabla^2 \vec{A} = \vec{J}$ where ρ is the electric charge density and $\vec{J}$ is the electric current density.

(ii) Show that the electric current density four-vector $J = (\rho, \vec{J})$ satisfies a continuity equation $\frac{\partial \rho}{\partial t} + \vec{\nabla}.\vec{J} = 0$. Confirm that this equation governs charge conservation.

QUESTION 2 (20 MARKS)

(a)(i) Write down Maxwell's equations in a material medium containing electric charge density ρ and current density $\vec{J}$.

(ii) Show that in the material medium, the electromagnetic field energy satisfies an energy transfer equation $\frac{\partial \varepsilon}{\partial t} + \vec{\nabla}.\vec{S} = -\vec{E}.\vec{J}$ where ε is the energy density, $\vec{S}$ is the Poynting vector and $\vec{E}$ is the electric field intensity.

(b) Consider a system of charge density ρ moving with velocity $\vec{v}$. If the mass of the system is

M, the Lorentz force equation in the medium takes the form $M\frac{d\bar{v}}{dt} = \rho\bar{E} + \rho\bar{v}\times\bar{B}$. The

density is $\bar{J} = \rho\bar{v}$.

(i) By taking the dot product with velocity $\bar{v}$, convert the Lorentz force equation into the energy transfer equation $\frac{d}{dt}\left(\frac{1}{2}M\bar{v}^2\right) = \bar{E}.\bar{J}$

(ii) By combining the energy transfer equations in a(ii) and b(i) above and noting $\frac{d}{dt} = \frac{\partial}{\partial t} + \bar{v}.\bar{\nabla} \Rightarrow \frac{d}{dt}\left(\frac{1}{2}Mv^2\right) = \frac{\partial}{\partial t}\left(\frac{1}{2}Mv^2\right)$, show that the total field and system energy is conserved.

QUESTION 3 (20 MARKS)

(a) (i)Solve the wave equation $\frac{1}{c^2}\frac{\partial^2\bar{E}}{\partial t^2} - \nabla^2\bar{E} = 0$ for an electric field intensity in vacuum and express the general solution in the form $\bar{E}(\bar{r},t) = \bar{E}_0\cos(\omega t + \theta_1)\cos(\bar{k}.\bar{r} + \theta_2)$ where all symbols have usual meanings to be specified.

(ii) Determine the magnetic field intensity $\bar{B}(\bar{r},t)$ of the electromagnetic field subject to Maxwell's equation $\bar{\nabla}\times\bar{E} = -\frac{1}{c}\frac{\partial\bar{B}}{\partial t}$

(b) Calculate the energy density and pointing vector of this electromagnetic field in vacuum.

QUESTION 4 (20 MARKS)

(a) Using the contravariant and covariant notation for four-vectors to define $\partial_\mu = \frac{\partial}{\partial x^\mu}$, $A_\mu = (A_0 - \bar{A})$; $A_0 = A^0$, $\mu = 0,1,2,3$, obtain a full expansion of the electromagnetic field tensor $F_{\mu\omega}$ in terms of the electric and magnetic field intensities in 4×4 matrix form.

(b)(i) Show that the field tensor $F_{\mu\omega}$ can be expressed in terms of Lorentz boost and rotation matrices satisfying standard algebra of the Lorentz group.

(ii) Determine the Lorentz pure boost element along the x-axis using the boost matrix K_x associated with $F_{\mu\nu}$.

(iii) Obtain the Lorentz boost transformation of the event four-vector X along the x-axis using boost element along the x-axis calculated in (ii) above.

QUESTION 5 (20 MARKS)

(a) Obtain the general solutions of Maxwell's equations for prescribed sources and evaluate the retarded fields.

(b) Calculate the electromagnetic dipole radiation arising from the retarded fields.