MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2015/2016

FIRST YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF MASTER OF SCIENCE IN PHYSICS

MAIN CAMPUS

SPH 803: QUANTUM MECHANICS

Date: 18th December, 2015 Time: 9.00 - 12.00pm

INSTRUCTIONS:

- Answer any THREE Questions.

SPH 803: QUANTUM MECHANICS

Answer any **THREE** questions.

QUESTION 1 (20 MARKS)

(a) (i) Explain briefly the Schroedinger and Heisenberg pictures of quantum mechanics.

(ii) Suppose a wave function in the Schroedinger picture is obtained in the form

$\Psi(\bar{r},t)=e^{-\frac{i}{\hbar}Ht}\psi(\bar{r})$, where H is the Hamiltonian. Given a Hermitian operator $\hat{O}$ in the Schroedinger picture, derive the form of the time evolving operator $\hat{O}(t)$ in the Heisenberg picture.

(iii) Derive the Heisenberg equation for the operator $\hat{O}(t)$.

(iv) Transform the time-dependent Schroedinger equation $i\hbar\frac{\partial\psi}{\partial t}=H\psi; H=H_0+H_I$ into the equivalent equation in the interaction picture, given H_0 as unperturbed Hamiltonian and H_I as the interaction Hamiltonian.

(b) (i) Explain briefly the mathematical and Physical origins of Dirac's ket and bra representation of quantum states.

(ii) The energy levels of a quantized linear harmonic oscillator are represented by a number state vector $|n\rangle, n=0,1,2...$ Using the annihilation operator $\hat{a}$ and the creation operator a^+ as the basic operators for the quantized oscillator, calculate the uncertainty product in the measurements of the quadrature components x_1, x_2 defined by $\hat{a}=\hat{x}_1+i\hat{x}_2$; $\hat{a}^+=\hat{x}_1-i\hat{x}_2$ if the oscillator is in the quantum state $|n\rangle$.

QUESTION 2 (20 MARKS)

A three-level atom in an electromagnetic field is described by a general wave function $\Psi(\bar{r},t)=c_1(t)\psi_1(\bar{r})+c_2(t)\psi_2(\bar{r})+c_3(t)\psi_3(\bar{r})$ where $c_j(t), j=1,2,3$ described by stationary wave functions $\psi_j(\bar{r})$. If the Hamiltonian of the system is $H = H_0 + H_I$ where H_0 is the free atom Hamiltonian satisfying eigenvalue equation $H_0\psi_j(\bar{r}) = E_j\psi_j(\bar{r})$ and H_I is the atom-field interaction.

(a) (i) Write down the Schroedinger equation for the interacting atom field system.

(ii) Derive the time evolution equations for probability amplitudes in the matrix form $i\hbar\frac{dC}{dt} = \overline{H}C$ where the amplitude matrix C and time evolution generating Hamiltonian $\overline{H}$ are to be defined.

(b) If all elements of $\overline{H}$ vanish, except $E_1 \neq 0, E_2 \neq 0, E_3 \neq 0, H_{12} = H_{21} \neq 0$ all of which are constant, solve the time evolution equation in a(ii) above explicitly.

QUESTION 3 (20 MARKS)

Consider the time-independent Schroedinger equation $\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\bar{r})\right)\psi(r) = E\psi(\bar{r})$ as a scattering problem for a particle of mass m in a scattering potential $V(\bar{r})$.

(a) By applying a factorization procedure, determine the scattering amplitude.

(b) Calculate the scattering cross-section in a screened Coulomb potential and use the result to obtain the scattering potential for a Coulomb potential without screening.

QUESTION 4 (20 MARKS)

A relativistic particle of rest mass m_0 linear momentum $\bar{p}$ and total energy E is governed by an energy conservation equation $E^2 = m_0^2c^4 + p^2c^2; p^2 = \bar{p}.\bar{p}$ where $p = |\bar{p}|$ and c is the speed of light.

a) (i) Show that the energy conservation equation can be expressed in an alternative four-vector inner product form $(E - m_0c^2, -\bar{p}c)(E + m_0c^2, \bar{p}c)(E + m_0c^2, \bar{p}c) = 0$.

(ii) Introduce two partner energy four-vectors P and $\overline{P}$ with temporal components $E + m_0c^2$ and $E - m_0c^2$ respectively to express the energy conservation equation in covariant form.

(b) (i) By applying the Dirac four-vector matrix algebraic relation $\gamma_\mu\gamma^\mu = \gamma^\mu\gamma_\mu = 4$ and introducing the energy and momentum quantum operators according to $\hat{E} = i\hbar\frac{\partial}{\partial t}$;

$\hat{p} = -i\hbar\vec{\nabla}$, use the covariant form $\overline{P}_{\mu}P^{\mu} = \overline{P}^{\mu}P_{\mu} = 0$ or the alternative form

$P_{\mu}\overline{P}^{\mu} = P^{\mu}\overline{P}_{\mu} = 0$ to obtain the standard Dirac t equations for the electron and its antiparticle, the positron.

(ii) Obtain the general solution for the Dirac equation for a free electron.

QUESTION 5 (20 MARKS)

The spin-up and spin-down state vectors $|u\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; |d\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(a) (i) Calculate the orthornomalization relations for the spin-state vectors $|u\rangle$ and $|d\rangle$ by evaluating the inner products $\langle j|k\rangle, j, k = u, d$

(ii) Determine the matrix forms of the Pauli spin operators

$$\hat{S}_z = \frac{\hbar}{2}(|u\rangle\langle d| - |d\rangle\langle u|); \hat{S}_+ = \hbar|u\rangle\langle d|$$
$$\hat{S}_- = \hbar|d\rangle\langle u|$$

(iii) Calculate the mean values of the operators $\hat{S}_z, \hat{S}_+, \hat{S}_-$ in the spin-up and spin down states $|u\rangle, |d\rangle$ of the electron.

(b) The dynamics of an electron in a static magnetic field is generated by Hamiltonian expressed in the form $H = g(\hat{S}_+ + \hat{S}_-)$ where g is a constant. If the electron is initially in spin state $|u\rangle$, calculate the probability of the transition to spin state $|d\rangle$ after time t.