MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2015/2016

FIRST YEAR SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF MASTER OF SCIENCE IN PHYSICS

MAIN CAMPUS

SPH 812: QUANTUM OPTICS I

Date: 12th May, 2016 Time: 2.00 - 5.00 pm

INSTRUCTIONS:

- Answer ANY THREE questions.
- Each question carries 20 marks.

SPH 812 : QUANTUM OPTICS I

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Question One

(a) **(i)** Write down the time-dependent Schroedinger equation for the wave function $\psi(t,\vec{r})$ of a particle interacting with an external field governed by total Hamiltonian $H = H_0 + H_I$, where all symbols have usual meanings

(ii) Transform the time-dependent Schroedinger in (i) above into the equivalent time evolution equation in the interaction frame, specifying the form of the resultant interaction Hamiltonian

(iii) Assuming the resultant interaction Hamiltonian in the interaction frame is time-independent, determine the time evolution operator $U_I(t)$ in the interaction frame

(iv) Use the result in (iii) above to determine the general time evolution operator $U(t)$ of the particle in the original frame

(b) (i) The total Hamiltonian of a quantized linear harmonic oscillator driven by an external classical field takes the form

$$H = \hbar\omega\left(\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\right) + \alpha e^{-i\omega t}\hat{a}^{\dagger} + \alpha^{*}e^{i\omega t}\hat{a}$$

where all symbols have usual meanings. Identify the components H_0 and H_I of this Hamiltonian

(ii) Show that the resultant interaction Hamiltonian of the oscillator in the interaction frame is time-independent

(iii) Determine the general time evolution operator $U(t)$ of the driven quantized oscillator under the Hamiltonian H given in (b)(i) above

(iv) Show that $U(t)$ obtained in (b)(iii) above can be interpreted as a displacement operator and state the nature of the resulting quantum states of the oscillator in the interaction generated by the Hamiltonian H in (b)(i)

Question Two

(a) **(i)** The general wave function of a two-level atom interacting with an external classical electromagnetic field is obtained in the form

$\Psi(t,\vec{r}) = c_0(t)\psi_0(\vec{r}) + c_1(t)\psi_1(\vec{r})$

If the Hamiltonian takes the form $H = H_0 + H_I$, derive the time evolution equations for the level probability amplitudes $c_0(t)$ and $c_1(t)$. All symbols have usual meanings.

(ii) By writing the time evolution equations for $c_0(t)$ and $c_1(t)$ in matrix form, determine the spin state time evolution generating Hamiltonian matrix , expressed in terms of Pauli matrices as appropriate

(b) (i) If the external classical electromagnetic field is static, solve the matrix equation in (a)(ii) above to determine the general form of the probability amplitudes $c_0(t)$, $c_1(t)$ if the atom is initially in the ground state $\psi_0(\vec{r})$

(ii) Calculate the probability to be in the excited state $\psi_1(\vec{r})$ at any time t during interaction with the static external field

(iii) Write down the general form of the wave function $\Psi(t,\vec{r})$ describing the states of the atom interacting with the static classical electromagnetic field under the initial condition specified in (b)(i) above.

Question Three

(a) **(i)** The general spin state vector $|\psi(t)\rangle = a(t)|u\rangle + b(t)|d\rangle$ for an electron under the action of a harmonically time varying magnetic field is governed by time evolution equation

$$i\hbar \frac{d|\psi\rangle}{dt} = H|\psi\rangle$$

where the Hamiltonian takes the form

$$H = \Delta\sigma_z + g(e^{-i\Omega t}\sigma_+ + e^{i\Omega t}\sigma_-)$$

All symbols have usual meanings and Δ , g , Ω

are constants.

Transform the spin state time evolution equation to an

appropriate rotating frame

(ii) Determine the general exact time evolution operator $U(t)$ governing the electron spin state dynamics in the original frame

(iii) Obtain the time evolving spin-up and spin-down coherent state vectors $|u;t\rangle$ and $|d;t\rangle$

(iv) Calculate the mean value of the Pauli spin operator $\hat{S}_z = \frac{\hbar}{2}(|u\rangle\langle u| - |d\rangle\langle d|)$ in the spin-up and spin-down coherent states $|u;t\rangle$ and $|d;t\rangle$.

(b) Calculate the general electron spin state density operator $\hat{\rho}(t) = |\psi(t)\rangle\langle\psi(t)|$ and show that $Tr\,\hat{\rho}(t) = 1$

Question Four

In the full quantum interaction between a two-level atom and a quantized external electromagnetic field mode, electron spin dynamics governed by the time evolution equation

$$i\hbar \frac{d|\psi\rangle}{dt} = H|\psi\rangle$$

is generated by a Jaynes-Cummings model Hamiltonian of the form

$$H = \Delta\sigma_z + \alpha\hat{a}\sigma_+ + \alpha^*\hat{a}^\dagger\sigma_-$$

where σ_z , $\sigma_\pm$ are Pauli matrices, $\hat{a}$, $\hat{a}^\dagger$ are the quantized field mode annihilation and creation operators, while Δ , α are constants, with α^* the complex conjugate of α

(a) **(i)** Obtain a simple solution of the spin equation

(ii) Evaluate the resulting Jaynes-Cummings time evolution operator explicitly

(b) (i) Determine the general electron-photon spin state vector $|\psi(t)\rangle$ describing the Jaynes-Cummings mode of dynamics if the photon is initially in a number state $|n\rangle$ and the electron is initially in a superposition of spin-up and spin-down state $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|u\rangle + |d\rangle)$

(ii) Calculate the mean values of the Pauli spin operators $\hat{S}_j$, $j = x, y, z$ in the general Jaynes-Cummings spin state $|\psi(t)\rangle$ obtained in (b)(i) above and use the results to explain the geometrical distribution of the electron spin states

Question Five

(a) (i) Explain Einstein's theory of spontaneous and stimulated emission and determine the corresponding A and B coefficients for transitions between two energy levels in an atom

(ii) Derive the population inversion rate equation for two-level laser system. Obtain a solution under steady state conditions

(b) The rate of spontaneous emission of a homogeneously broadened laser transition at $10.6\mu m$ is $A_{21} = 0.34s^{-1}$, while its linewidth is $\Delta\nu = 1GHz$. The degeneracies of the lower and upper levels are $g_1 = 41$ and $g_2 = 43$, respectively

(i) Calculate the stimulated emission cross-section at line centre

(ii) Calculate the population inversion to obtain a gain coefficient of $5m^{-1}$