MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2015/2016

FIRST YEAR SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF MASTER OF SCIENCE IN PHYSICS

MAIN CAMPUS

SPH 820: QUANTUM OPTICS II

Date: 13th May, 2016 Time: 2.00 - 5.00 pm

INSTRUCTIONS:

- Answer ANY THREE questions.
- Each question carries 20 marks.

SPH 820 : QUANTUM OPTICS II

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Question One

The Hamiltonian generating the dynamics of a quantized electromagnetic mode interacting with an external classical current is obtained in the form

$$H = \hbar\omega\hat{a}^{\dagger}\hat{a} + f(t)\hat{a}^{\dagger} + f^{*}(t)\hat{a}$$

where $\hat{a}$, $\hat{a}^{\dagger}$ are the field mode annihilation and creation operators, while $f(t)$ and its complex conjugate $f^{*}(t)$ represents the arbitrarily time varying external classical current.

(a) (i) Solve the Heisenberg equations of motion for the field mode annihilation and creation operators $\hat{a}(t)$, $\hat{a}^{\dagger}(t)$
(ii) Show that the time evolution of the annihilation and creation operators determined in (i) above is generated by a time evolution operator obtained as a general displacement operator

(iii) If the field mode was initially in the number ground state $|0\rangle$, establish that the general time evolving state vector of the classically driven quantized field mode is a coherent state vector $|\bar{\alpha}(t)\rangle$, where the interaction variable $\bar{\alpha}(t)$ may have been defined in (ii)

(b) (i) Show that the coherent state vector $|\bar{\alpha}(t)\rangle$ is expressible as a superposition of the photon number state vectors $|n\rangle$

(ii) Use the result obtained in (b)(i) to derive a general relation for normalization and non-orthogonality of coherent state vectors

(iii) Determine the uncertainty in the measurement of the mean value of the field mode photon number $\hat{n}(t) = \hat{a}^{\dagger}(t)\hat{a}(t)$

Question Two

A parametric oscillation process is generated by a time-independent Hamiltonian of the form

$$H = \hbar\omega_a \hat{a}^{\dagger}\hat{a} + \hbar\omega_b \hat{b}^{\dagger}\hat{b} + f\hat{a}\hat{b}^{\dagger} + f^{*}\hat{b}\hat{a}^{\dagger}$$

where ω_a , $\hat{a}$, $\hat{a}^{\dagger}$ and ω_b , $\hat{b}$, $\hat{b}^{\dagger}$ are signal and idler photon angular frequency, annihilation and creation operators, respectively, while f and its complex conjugate f^*, is the time-independent classical pump photon coupling parameter.

(a) **(i)** Derive the Heisenberg equations for the signal and idler photon annihilation operators

(ii) Apply a matrix method or any other effective method to obtain the general solutions of the time evolution equations derived in (i) above

(iii) Show that the total signal and idler photon number operator $\hat{n}(t)=\hat{a}^{\dagger}(t)\hat{a}(t)+\hat{b}^{\dagger}(t)\hat{b}(t)$ in the parametric oscillation process is conserved

(b) By calculating the uncertainties in the measurements of the mean values of signal and idler photon quadrature components

$\hat{a}_1(t)=\frac{1}{2}\left(\hat{a}(t)+\hat{a}^{\dagger}(t)\right)$; $\hat{b}_1(t)=\frac{1}{2}\left(\hat{b}(t)+\hat{b}^{\dagger}(t)\right)$

when the initial signal-idler photon state is the number state $|nm\rangle=|n\rangle|m\rangle$, determine the (quantum noise) amplification and attenuation properties of the parametric oscillation process. Take the initial signal photon state vector to be $|n\rangle$ and the initial idler photon state vector to be $|m\rangle$

Question Three

A non-degenerate parametric down-conversion process is governed by the time evolution equations

$$i\hbar\frac{d\hat{a}}{dt} = \hbar\omega_a\hat{a} + if\hat{b}^\dagger \qquad ; \qquad i\hbar\frac{d\hat{b}^\dagger}{dt} = -\hbar\omega_b\hat{b}^\dagger + if^*\hat{a}$$

where ω_a , $\hat{a}$ are the signal photon angular frequency and annihilation operator, while ω_b , $\hat{b}^\dagger$ are the idler photon angular frequency and creation operator

(a) **(i)** Obtain the general solutions of the time evolution equations, given that the angular frequencies and the classical pump photon coupling parameter f and its complex conjugate f^* are constant

(ii) Show that in the down-conversion process, the signal-idler photon number operator difference $\hat{n}_a(t) - \hat{n}_b(t) = \hat{a}^\dagger(t)\hat{a}(t) - \hat{b}^\dagger(t)\hat{b}(t)$ is conserved

(iii) Prove that signal and idler photons are produced at the same rate in the non-degenerate down-conversion process

(b) Calculate the signal-idler photon correlation function $\overline{a(t)b(t)} = \langle nm|\hat{a}(t)\hat{b}(t)|nm\rangle$ for signal-idler photon initial number state vector $|nm\rangle = |n\rangle|m\rangle$, where $|n\rangle$ is the signal photon initial number state vector and $|m\rangle$ is the idler photon initial number state vector

Question Four

The Hamiltonian for a degenerate parametric amplifier is obtained in the form

$$H = \hbar\omega\hat{a}^{\dagger}\hat{a} + \frac{i}{2}\left(g\hat{a}^{\dagger 2} - g^{*}\hat{a}^{2}\right)$$

where $\hat{a}$, $\hat{a}^{\dagger}$ are annihilation and creation operators and ω , g , g^{*} are constant

(a) **(i)** Determine the general form of the time evolving annihilation and creation operators $\hat{a}(t)$, $\hat{a}^{\dagger}(t)$ of the amplifier

(ii) By calculating the uncertainties in the measurements of mean values of the quadrature components

$$\hat{a}_1(t) = \frac{1}{2}\left(\hat{a}(t) + \hat{a}^{\dagger}(t)\right) \quad ; \quad \hat{a}_2(t) = -\frac{i}{2}\left(\hat{a}(t) - \hat{a}^{\dagger}(t)\right)$$

in initial number state $|n\rangle$, determine the general squeezing property of the degenerate parametric amplifier

(b) Establish that under strong-coupling condition $g \gg \hbar\omega$, for real $g = g^{*}$, the uncertainty in one quadrature component grows exponentially, while the uncertainty in the other quadrature component decays exponentially such that the initial quadrature uncertainty product is preserved

Question Five

In an optical process in which a photon generated from a high intensity laser source interacts with atoms in a non-linear crystal lattice creates a signal and idler photons with respective annihilation operators $\hat{a}$ and $\hat{b}$, the signal-idler photon pair polarization state dynamics is governed by the time evolution equation

$$i\hbar\frac{d\hat{A}}{dt} = H\hat{A}$$

where the signal-idler photon polarization operator vector $\hat{A}$ and Hamiltonian matrix H take the form

$$\hat{A} = \hat{a}|h\rangle + \hat{b}|v\rangle \quad ; \quad H = \Delta\sigma_z + \frac{1}{2}\left(\alpha\sigma_+ + \alpha^*\sigma_-\right)$$

All symbols have usual meanings. The basic horizontal and vertical polarization state vectors $|h\rangle$, $|v\rangle$ defined in usual form satisfy standard orthonormalization relations

(a) Taking the Hamiltonian matrix to be time-independent, determine the general signal-idler photon polarization state time evolution operator $U(t)$ in explicit form

(b) (i) Given initial signal-idler photon number state vector $|nm\rangle$ defined as usual, determine the initial signal-idler photon polarization state vector $|\psi(0)\rangle$

(ii) By calculating the inner product $\langle\psi(0)|\psi(0)\rangle$, prove that signal and idler photons cannot both be in an initial vacuum state $|00\rangle$, i.e , $|\psi(0)\rangle \neq |00\rangle$

(c) (i) Determine the general time evolving signal-idler photon horizontal and vertical polarization coherent state vectors $|h;t\rangle$ and $|v;t\rangle$ in this non-linear optical process

(ii) Calculate the mean values of the basic polarization operator $\hat{S}_z = \frac{1}{2}(|h\rangle\langle h| - |v\rangle\langle v|)$ in the time evolving horizontal and vertical polarization coherent state vectors $|h;t\rangle$ and $|v;t\rangle$. Give an interpretation of the results