MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2015/2016

FIRST YEAR SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF MASTER OF SCIENCE IN PHYSICS

MAIN CAMPUS

SPH 822: MATHEMATICAL METHODS FOR PHYSICS

Date: 11th May, 2016 Time: 2.00 - 5.00 pm

INSTRUCTIONS:

- Answer ANY THREE questions.

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SPH822: MATHEMATICAL METHODS FOR PHYSICS

Answer any **three Questions**

Q1. a) Solve the differential equation (5M

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$$

b) Solve $y'' - 2y' + y = 2cosx$ by use of successive integration (5M

c) Find the Laurent series of (10M

$$f(z) = \frac{1}{z(z-2)^3}$$

About the singularities $z = 0$ and $z = 2$ (separately). Hence verify that $z = 0$ is a pole of order 1 and $z = 2$ is a pole of order of 3, and find the residue of $f(z)$ at each pole.

Q2. a) Find the square root of i (8M

b) Expand $\frac{1}{(1-z)}$ in a Taylor's series about $Z_o = i$ and find the radius of convergence (12M

Q3. a) Using the residue theorem, evaluate:

i) $I = \int_0^{2\pi} \frac{d\theta}{5+4\cos\theta}$ (4M

ii) $I = \int_{-\infty}^{\infty} \frac{sinxdx}{x}$ (3Mks

iii) $I = \int_{0}^{\infty} \frac{dx}{(x^2+a^2)^4}$, where a is real (4Mks

b) Evaluate the residues of the following functions: (9Mks)

i) $\frac{Z^2-1}{Z^2+1}$ ii) $\frac{1}{Z^2+4Z+1}$

Q4. a) Use the method of separation of variables to solve a two dimensional Laplace equation.

$$\frac{\partial^2 U}{\partial t^2} + \frac{\partial^2 U}{\partial x^2} = 0$$

b) Using the method of direct integration solve the following equation (10Mk

$$\frac{\partial^2 U(x,y)}{\partial x \partial y} + x^2 y = 0$$

Subject to the conditions: $z(x,0) = x^2$ and

$z(1,y) = \cos y$

Q5. You have a slab of material of thickness L and at a uniform temperature T_o (20Mk
The side at $x = L$ is insulated so that heat can't flow in or out of that surface. Plunge the other side into ice water at temperature $T = 0$ and find the temperature inside at later time. The boundary condition on the $x = 0$ surface is $T(0,t) = 0$.

i) Separate variables and find the appropriate separate solutions for these boundary conditions.

ii) Are the separate solutions orthogonal?

iii) When the lowest order term has dropped to where its contribution to the temperature at $x = L$ is $T_o/2$, how big is the next term in the series?

iv) Sketch the temperature distribution in the slab at that time.