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**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTURIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTURIAL**

**4th YEAR 1st SEMESTER 2015/2016 ACADEMIC YEAR**

**MAIN REGULAR**

**COURSE CODE: SAS 401**

**COURSE TITLE: FURTHER DISTRIBUTION THEORY**

**EXAM VENUE: STREAM: (Bsc. Actuarial Science)**

DATE: EXAM SESSION: SEP-DEC 2016

TIME: 2.00 HOURS

**Instructions:**

1. **Answer questions one and any other two.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

**QUESTION ONE (30 MARKS)**

(a) Show that if $X\_{1}$ and $X\_{2}$ are two independent Poisson variates with parameters $θ\_{1}$ and $θ\_{2}$ respectively, then the conditional distribution of $X\_{1}$ given $X\_{1}+X\_{2}=n$ is a binomial distribution. (5 marks)

(b) Given $f\left(x\right)=2\left(1-x\right); 0<x<1$.

Find the distribution of $Y=\left(1-X\right)^{2}$ (5 marks)

(c) The probability generating function of a random variable $X$is given by

$p\left(x\right)= \left(\frac{1+S}{2}\right)^{n}$

Find the probability function of $X$. (5marks)

(d) Let $X\_{1}, X\_{2, \cdots , }X\_{k}$ be $k $ independent random variables. The probability function of $X\_{i}$ ($i=1, 2, \cdots ,k$) is

$$p\left(x\_{i}\right)=p\left(1-p\right)^{x};x=1,2, \cdots $$

Find the probability generating function of $\sum\_{i=1}^{n}X\_{i}$ and hence its probability functions. (5marks)

(e) Show that Normal distribution is a particular case of $χ^{2}$ with $n-1$*df*. (5 marks)

(f) Let $X\_{1}, X\_{2, \cdots , }X\_{n}$ be a random sample drawn from a population having density

$$f\left(x\right)=1 ;0<x<1$$

Find the distribution of $x\_{(r)}$. Also, find the mean and variance of this distribution. (5 marks)

**QUESTION TWO(20 Marks)**

Four hundred samples of each of 1 c.c. of a liquid were investigated to study the number of Bacteriapresent in each sample. The maximum number of Bacteria was found 8 in the samples. Below is a given number of Bacteria $(x)$ in different samples $(f)$

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Number of Bacteria (x) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Total |
| Number of samples (f) | 107 | 142 | 92 | 45 | 8 | 4 | 2 | 0 | 0 | 400 |

Fit a Poisson distribution using the above data. (20 marks)

**QUESTION THREE(20 Marks)**

Construct the following distributions using power series method

1. Poisson (5 marks)
2. Binomial (5 marks)

Hence obtain

1. Expectations (5 marks)
2. Variances (5 marks)

**QUESTION FOUR(20 Marks)**

1. Suppose the *pdf* of Gamma distribution with one parameter is

$$g\left(λ\right)= \frac{e^{-λ}λ^{α-1}}{Γ\_{α}}; λ>0, α>0$$

And the *pmf* of Poisson distribution provided by

$$f\left(x\right)= \frac{e^{-λ}λ^{x}}{x!}; x=0, 1,2, \cdots $$

1. Find mixed distribution of Poisson and Gamma (8 marks)
2. What is the resulting distribution from the mixture? (2 marks)
3. Assuming that a two parameter Gamma distribution

$$g\left(λ\right)= \frac{β^{α}e^{-βλ}λ^{α-1}}{Γ\_{α}}; λ>0, α>0, β>0 $$

1. Show that the mixed Poisson distribution and the two parameter Gamma result to a negative binomial distribution. (10 marks)

**QUESTION FIVE(20 Marks)**

If $X$ and $Y$ are two independent standard normal variates, find the distribution of

1. $X^{2}$ (7 marks)
2. $X^{2}+Y^{2}$ (7 marks)
3. $\frac{X^{2}}{Y^{2}}$ (6 marks)