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**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTURIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTURIAL**

**4th YEAR 2nd SEMESTER 2016/2017ACADEMIC YEAR**

**MAIN REGULAR**

**COURSE CODE: SAS 402**

**COURSE TITLE: BAYESIAN INFERENCE AND DECISION THEORY**

**EXAM VENUE: STREAM: (Bsc. Actuarial Science)**

DATE: EXAM SESSION: SEP-DEC 2016

TIME: 2.00 HOURS

**Instructions:**

1. **Answer questions one and any other two.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

**QUESTION ONE (30 MARKS)**

(a) Suppose $p\left(n,π\right)=\left(\genfrac{}{}{0pt}{}{n}{x}\right)π^{x}(1-π)^{n-x}$ which is the likelihood distribution with prior provided by $π\~Be(α,β)p\left(π\right)= \frac{Γ\_{\left(α+β\right)}}{Γ\_{α}Γ\_{β}}π^{α-1}(1-π)^{β-1}$ for $π\in \left[0,1\right]$

Find

1. The posterior distribution $p\left(π|X\right)$ (3 marks)
2. The posterior mean $\left(E\left(π|X\right)\right)$ and variance $\left(var\left(π|x\right)\right)$ (2 marks)

(b) Given a CDF $F\left(a\right)=pr(Y \leq a)$, formally

1. show that $pr(Y $>$a)=1-F\left(a\right)$ using only the axioms of probability and the definition of a CDF (3 marks)
2. Show that $pr(a<Y \leq b)=F\left(b\right)-F\left(a\right)$ (3 marks)

(c) Suppose the outcome $y$ of genetics experiment has a binomial (4, $θ$) distribution with unknown parameter $θ$ . Also, suppose that it is known that $θ$ is either $\frac{1}{2}$ or $\frac{1}{4}$ , and we have no reason to believe one possibility over the other. Compute $pr\left(θ= \frac{1}{2}\right|Y=2)$ (3marks)

(d) The definition of the expectation of a random variable $y$ is $E\left(Y\right)= \sum\_{y\in Y}^{}yp(y)$. Suppose $Y\~binomial(n, \frac{1}{4})$

Compute $E(y)$ and $E(y^{2})$ (3marks)

Compute $var(y)$ from the expectation you just computed. (2 marks)

(e) Let $a$ and $b$ be any constant numbers. Using the definition of expectations, derive formulae for

1. $E(aY+b)$ (3 marks)
2. $var(aY+b)$ (5 marks)

(f) Let $x$ be a Binomial random variable with parameters $n$ and $p$, where

$$f\_{1}\left(p\right)=1, 0<p<1$$

Find the Bayes estimator of $p$ (3 marks)

**QUESTION TWO(20 Marks)**

1. A machine is built to make mass-produce items. Each item made by the machine has a probability $p $of being defective. Given the value of $p$, the items are independent of each other. Because of the way in which the machines are made, $p$ could take one of several values. In fact $p=\frac{X}{100}$ where x has a discrete uniform distribution on the interval $\left[0,5\right]$. The machine is tested by counting the number of items made before a defective is produced. Find the conditional probability distribution of x given that the first defective item is the thirteenth to be made. (10 marks)
2. There are five machines in a factory. Of these machines, three are working properly and two are defective. Machines which are working properly produce articles each of which has independently a probability of 0.1 of being imperfect. For the defective machines, this probability is 0.2. A machine is chosen at random and five articles produced by the machine are examined. What is the probability that the machine chosen is defective given that, of the five articles examined, two are imperfect and three are perfect?(10 marks)

**QUESTION THREE(20 Marks)**

1. Let $x\_{1}, \cdots ,x\_{n}$ be a random sample of n observations from Bernoulli population with parameter $θ$. Find the Bayes estimator of $θ.$ (10 marks)
2. Let $x\_{1}, \cdots ,x\_{n}$ be a random sample of $n$ observations from a Poisson population with parameter $θ$ such that $f\_{1}\left(θ\right)= e^{-θ}, θ>0$ and $f\_{1}\left(θ\right)= \frac{1}{α}e^{^{-θ}/\_{α}}, θ>0$. Using quadratic loss function find Bayes estimator of $θ$. (10 marks)

**QUESTION FOUR(20 Marks)**

1. In Mau forest area there may be wild Lion. At a particular time the number x of Lions can be between 0 and 5 with

$prob\left(X=x\right)= \left(\genfrac{}{}{0pt}{}{5}{x}\right)0.6^{x}0.4^{n-x}$ ($x=0, \cdots , 5$)

A survey is made but the Lion is difficult to spot and, given that the number present is $x$, the number $y$ observed has a probability distribution with

$$prob\left(Y=y|X=x\right)= \left\{\begin{array}{c}\left(\genfrac{}{}{0pt}{}{x}{y}\right)0.3^{y}0.7^{x-y} ; \left(0\leq y\leq x\right)\\0; (x<y)\end{array}\right.$$

Find the conditional probability distribution of x given that y=2. (10 marks)

1. The quadratic loss function in estimating $θ$ in $f(x,θ)$; $θ\in S$ is $L\left[θ, W(t)\right]= \left(θ-W\right)^{2}$. Show that if $θ$ is estimated with this loss function, then the Bayes estimator is $W\left(t\right)=E\left[θ|X=x\right]$

 (10 marks)

**QUESTION FIVE(20 Marks)**

1. A Bayesian posterior distribution is obtained by mixing Gamma to a Poisson distribution. Suppose the *pdf* of Gamma distribution with one parameter is

$g\left(λ\right)= \frac{e^{-λ}λ^{α-1}}{Γ\_{α}}; λ>0, α>0$ , and the conditional *pmf* of Poisson distribution provided by

$$f\left(λ|x\right)= \frac{e^{-λ}λ^{x}}{x!}; x=0, 1,2, \cdots $$

1. Find mixed distribution of Poisson and Gamma (4 marks)
2. What is the resulting distribution from the mixture? (1 marks)
3. Provide the expectation and variance of the resulting posterior distribution. (5 marks)
4. Assuming that instead, a two parameter Gamma distribution was used such that

$$g\left(λ\right)= \frac{β^{α}e^{-βλ}λ^{α-1}}{Γ\_{α}}; λ>0, α>0, β>0 $$

1. Show that the mixed Poisson distribution and the two parameter Gamma result to a negative binomial posterior distribution. Verify the result of (a) when $β=1$. (10 marks)