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**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTURIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTURIAL**

**4th YEAR 2ndSEMESTER 2015/2016 ACADEMIC YEAR**

**MAIN REGULAR**

**COURSE CODE: SAS 408**

**COURSE TITLE: MULTIVARIATE METHODS**

**EXAM VENUE: STREAM: (Bsc Actuarial Science)**

DATE: EXAM SESSION: SEP-DEC 2016

TIME: 2.00 HOURS

**Instructions:**

1. **Answer questions one and any other two.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

**QUESTION ONE (30 Marks)**

(a) Let $X^{'}=\left[X\_{1}, X\_{2}\right]$ be a random vector with mean vector

$$μ^{'}\_{x}= \left[μ\_{1}, μ\_{2}\right]$$

And variance-covariance matrix

$$∑\_{x}= \left[\begin{matrix}σ\_{11}&σ\_{12}\\σ\_{21}&σ\_{22}\end{matrix}\right]$$

Find the means and covariance matrix for the linear combination

1. $Z\_{1}= X\_{1}- X\_{2}$ (3 marks)
2. $Z\_{2}= X\_{1}+ Z\_{2}$ (3 marks)

(b) Let $x$ be $N\_{3}(μ,$∑$)$ with

∑$= \left(\begin{matrix}4&1&0\\1&3&0\\0&0&2\end{matrix}\right)$

Prove whether or not the following are independent

1. $x\_{1}and x\_{2}$ (3 marks)
2. $(x\_{1},x\_{2}) $ and $x\_{3}$ (3 marks)

(c) For the matrix

A = $\left(\begin{matrix}3&1&1\\1&0&2\\1&2&0\end{matrix}\right)$

1. Could A be a covariance matrix? Explain (2 marks)
2. Obtain determinant of A and $A^{-1}$ (4 marks)
3. Compute the spectral decomposition of A (4 marks)

(d) Let random variables $\overline{x^{'}}=\left[x\_{1}, x\_{2}, x\_{3}\right]$ be distributed as $N\_{3}(\overline{μ},$∑$)$

$\overline{μ}=\left(\genfrac{}{}{0pt}{}{2}{\begin{array}{c}-1\\3\end{array}}\right)$ and $\sum\_{}^{}= \left[\begin{matrix}4&1&0\\1&2&1\\0&1&3\end{matrix}\right]$

Find the following

1. Correlation matrix of $\overline{x}$ (2 marks)
2. The distribution of $z=4x\_{1}-6x\_{2}+x\_{3}$ (3 marks)
3. The distribution of $z= \left(\genfrac{}{}{0pt}{}{x\_{1}-x\_{2}+x\_{3}}{2x\_{1}+x\_{2}-x\_{3}}\right)$ (3 marks)

**QUESTION TWO (20 Marks)**

1. Show that $\left|S\right|=0$ for

$$X= \left[\begin{matrix}1&4&4\\2&1&0\\5&6&4\end{matrix}\right]$$

1. Determine the degeneracy for (i) above.

**QUESTION THREE (20 Marks)**

Consider data matrix for n=3 for a bivariate distribution

$$X= \left[\begin{matrix}6&10&8\\9&6&3\end{matrix}\right]$$

$$\overbar{X }= \left[\begin{matrix}8\\6\end{matrix}\right]$$

Evaluate the observed $T^{2}$ for $\overline{μ}^{'}\_{0}$ = $\left[\begin{matrix}9&5\end{matrix}\right]$. What is the sampling distribution of $T^{2}$ in this case?

**QUESTION FOUR (20 Marks)**

Consider the covariance matrix

 $\sum\_{}^{}= \left[\begin{matrix}1&4\\4&100\end{matrix}\right]$

And the derived correlation matrix

$$ρ= \left[\begin{matrix}1&0.4\\0.4&100\end{matrix}\right]$$

Determine the principal components for $∑$ providing percentage of explained variability for each variate.

**QUESTION FIVE (20 Marks)**

1. Suppose

∑ (covariance matrix) = $\left[\begin{matrix}4&1&2\\1&9&-3\\2&-3&25\end{matrix}\right]$

Obtain standard deviation and population correlation matrix in the form of $˅^{^{1}/\_{2}}$and ρ respectively. (10 marks)

1. Given the deviation vectors

$e\_{1}= \left(\begin{matrix}2\\-3\\1\end{matrix}\right)$ and $e\_{2}= \left(\begin{matrix}-2\\0\\2\end{matrix}\right)$

Compute the sample variance-covariance matrix $S\_{n}$ and the sample correlation matrix $γ$ using geometric concept. (10 marks)