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**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF MASTER OF SCIENCE**

**IN PURE MATHEMATICS**

**1ST YEAR 1ST SEMESTER 2016/2017 ACADEMIC YEAR**

**MAIN CAMPUS**

**COURSE CODE: SMA 803**

**COURSE TITLE: FUNCTIONAL ANALYSIS I**

**EXAM VENUE: STREAM: (MSC PURE MATHEMATICS)**

**DATE: EXAM SESSION:**

**TIME:**

**Instructions:**

* **Answer ANY 3 questions**
* **Candidates are advised not to write on the question paper.**
* **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

 **QUESTION ONE (20 MARKS)**

 (i) Describe the terms: Metric space, Neigbourhood, Open set and Closure of a set.**[10 marks]**

 (ii) State and prove Weiener’s theorem. **[10 marks]**

**QUESTION TWO (20 MARKS)**

(i) Let (X, d ) be a metric space and Y a subset of X. Show that dYis a metric on Y and (Y, dY)

is a subspace of (X, d ). **[10 marks]**

(ii) Prove that if E is a normed space and M is a closed linear subspace of E such that E and

E/M areBanach spaces then E is a Banach space. **[10 marks]**

**QUESTION THREE (20 MARKS)**

 (i) Show that an arbitrary union of open sets is open.**[7 marks]**

 (ii) Show that a set is open if its complement is closed.**[7 marks]**

 (iii) Prove that a mapping T taking a Hilbert space H to its dual H\* defined by T(h) = Lh is an

anti-linear isometric bijection of H onto H\* if Lh takes H onto K.**[6 marks]**

**QUESTION FOUR (20 MARKS)**

(i) Prove that every Cauchy sequence is bounded but the converse need not be true. **[6 marks]**

 (ii) Prove that every convergent sequence is Cauchy but the converse need not be true. **[7 marks]**

 (iii) Using the axiom of choice, show the existence and uniqueness of unbounded linear functional.

**[7 marks]**

**QUESTION FIVE (20 MARKS)**

 (i) Construct an example of a series in a Hilbert space that converges unconditionally but not

absolutely. **[6 marks]**

(ii)State and prove Cantor’s intersection theorem. **[7 marks]**

(iii)State and prove Banach’s fixed point theorem. **[7 marks]**

**END**