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**University Examinations 2014/2015**

SECOND YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE, BACHELOR OF SCIENCE

**SMA 2203: NUMBER THEORY**

**DATE: DECEMBER 2014 TIME: 2 HOURS**

**INSTRUCTIONS:** *Answer question* ***one*** *and any other* ***two*** *questions*

**QUESTION ONE COMPULSORY (30 MARKS)**

1. Given a set S and a binary operation \* , define a group S under this operation (4 marks)
2. If a|b and b|d, prove that a|c (3 marks)
3. Use the euclidean algorithm to calculate the greatest common divisor of (1001, 1911, 9177) (5 marks)
4. (i) Form the first five intergers mk=p1p2........pk+1, where p1, p2.......are prime numbers from 2 in ascending order. Conclude that they (integers) are all prime (5 marks)

(ii) Show that m7 is not a prime number (1 mark)

1. (i) Define a linear diophantine equation (2 marks)

(ii)State any two possible solutions of the linear diophantine equation 2x+3y=2

(2 marks)

1. State the Fermat’s last theorem (2 marks)
2. Define a primitive Pythagorean triple (2 marks)
3. Solve 42x50 (mod 76) (4 marks)

**QUESTION TWO (20 MARKS)**

1. Calculate the least common multiple (lcm) of 236 and 136 (5 marks)
2. Show that 41 divides 220-1 (5 marks)
3. Solve the diophantine equation 738x+621y=45 (10 marks)

**QUESTION THREE (20 MARKS)**

1. Define a residue class modulo n (3 marks)
2. There are seven residue classes modulo 7. Name them (7 marks)
3. (i) Define the Pell’s equation (3 marks)

(ii) Show that the following rational approximations to are all “good” in the sense that $a^{2}-2b^{2}=\pm 1$

 (7 marks)

**QUESTION FOUR (20 MARKS)**

1. (i) Define a quadratic residue (mod n) (2 marks)

(ii) By considering 12, 22,...... 102 (mod 11) determine the quadratic residues (mod 11) and the non residues (mod 11) (10 marks)

1. Using the following definition that a belongs to the exponent k (mod n) if k is the smallest positive integer x such that $a^{x}≡1$(mod n); show that 5 belongs to the exponent 6 (mod 7). Conclude that the powers of 5 form residues (mod 7) (8 marks)

**QUESTION FIVE (20 MARKS)**

1. Determine whether 17 is a prime by deciding whether or not $16!≡-1$(mod 17)

(6 marks)

1. Arrange the integers 2,3,4.....,21 in pairs a and b with the property that $ab≡1$(mod 23)

(7 marks)

1. Show that $18!≡$-1 (mod 437) (7 marks)