JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTURIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTURIAL

4TH YEAR 2ND SEMESTER 2016/2017 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SAC 404

COURSE TITLE: COMPUTATIONAL FINANCE

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION 1 [COMPULSORY] [30 Marks]

(a) Write down expressions for the following derivative contracts in terms of the expiry time T, the price S_t of the underlying time t and the exercise price K;

(i) A European call option [**2 Marks**]

(ii)A European put option [**2 Marks**]

(b)State the six assumptions underlying the Black-Scholes market.[**6 Marks**]

(c) Define delta, gamma and vega of and individual derivative. [**4 Marks**]

(d)In a one-step binomial tree model it is assumed that the initial share price of 260 will either increase to 285 or decrease to 250 at the end of one year. Assume that the annual force of interest is 0.05 and that no dividends are payable. Calculate the price of a one year European call option with a strike price of 275,. [**8 Marks**]

(e)Consider a ten-month forward contract on a stock with a price of 50. We assume that the risk-free rate of interest(compounded continuously) is 8% per annum. We assume also that dividends of 0.75 per share are expected after 3 months, 6 months and 9 months.Calculate the forward price and the value of the forward contract if the strike price is 45. [**8 Marks**]

QUESTION 2 [20 Marks]

Consider a stock initially priced at S_0 and an option whose current price is f with the option lasting for time T.The stock price can go up to S_0u or down to S_0d;

(a) Show that the price of the option is given by

$$f = e^{-rT}(pf_u + (1-p)f_d)$$

where

$$p = \left(\frac{e^{rt} - d}{u - d}\right)$$

[8 Marks]

(b) Hence use the result in (a) to show that

$$E(S_T) = S_0 e^{rT}$$

where $E(S_T)$ is the expected stock price at time T. **[12 Marks]**

QUESTION 3 [20 Marks]

Consider a two-period binomial model for a non-dividend paying stock whose current price is $S_0 = 100$. Assume that:

- over each six-month period, the stock price can either move up by a factor $u = 1.2$ or down by a factor $d = 0.8$
- the continuously compounded risk-free rate is $r = 5\%$ per six-month period

(a) (i) Prove that there is no arbitrage in the market. **[2 Marks]**

(ii) Construct the binomial tree. **[2 Marks]**

(b) Calculate the price of a standard European call option written on the stock S with strike price $K = 100$ and maturity one year. Consider a special type of call option with strike price $K = 100$ and maturity one year. The underlying asset for this special option is the average price of the stock over one year, calculated as the average of the prices at times 0, 0.5 and 1 measured in years. **[8 Marks]**

(c) Calculate the initial price of this call option assuming it can be exercised only at time 1. **[8 Marks]**

QUESTION 4 [20 Marks]

The table below shows the portfolio of over-the-counter options on the Kenya shillings held by an investment bank.

Types of Option	Option Delta	Option Gamma	Option Vega	Size of Position
Call	0.5	2.2	1.8	-1,500
Call	0.8	0.6	0.2	-750
Put	-0.4	1.3	0.7	-3,000
Call	0.7	1.8	1.9	-750

(a) Calculate the delta, gamma and vega of the investment bank's portfolio.

[**9 Marks**]

(b) An exchange offers a traded option on Kenya shillings with the following parameters.

Delta:0.6

Gamma:1.5

Vega:0.8

Calculate the position in the traded option and in shillings to make the investment bank's portfolio both delta and gamma neutral. [**6 Marks**]

(c)Calculate the position in the traded option and in shillings needed to make the investment bank's portfolio both vega and delta neutral.

[**5 Marks**]

QUESTION 5 [20 Marks]

(a) The Black-Scholes formula for the value of a European call option on a non- dividend paying stock at time t can be written as

$$C = S\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2)$$

where

$$d_1 = \frac{ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

K=strike price

T=time of maturity

S=price of stock at time t

r=risk-free rate

σ=volatility

and $\Phi(.)$= cumulative distribution function of the standard normal distribution.

Using the Black-Scholes formula show that the call price C, is the maximum of $S - Ke^{-r(T-t)}$ or 0, depending on the strike price when σ tends to zero.

[12 Marks]

(b)Show that if d_1 and d_2 are as indicated in (a) above then

$$Se^{-(T-t)}e^{-\frac{1}{2}d_1^2} = Ke^{-r(T-t)}e^{-\frac{1}{2}d_2^2}$$

[8 Marks]