



W1-2-60-1-6

**JOMO KENYATTA UNIVERSITY  
OF  
AGRICULTURE AND TECHNOLOGY**  
**UNIVERSITY EXAMINATIONS 2014/2015**

**FOURTH YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF  
BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE**

**SMA 2432: DESIGN AND ANALYSIS OF SAMPLE SURVEYS**

**DATE: AUGUST 2015**

**TIME: 2 HOURS**

**INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER  
TWO QUESTIONS**

**QUESTION ONE (30 MARKS)**

a) Given that you have a population of  $N$  units and a sample of  $n$  units.  
Show that  $\bar{y}$  is an unbiased estimator of  $\bar{Y}$  [4 marks]

b) Consider a population of size  $N=3$ . The values of  $Y_i$ 's are 0, 2 and 1 for  $i = 1,2,3$  respectively. Consider the following two estimators of the population mean for the SRSWOR of size 2 from the population:

i)  $\bar{y}$  the sample mean

$$t = \begin{cases} t_1 = \frac{1}{2}y_1 + \frac{1}{2}y_2 \\ t_2 = \frac{1}{2}y_1 + \frac{2}{3}y_3 \\ t_3 = \frac{1}{2}y_2 + \frac{1}{3}y_3 \end{cases}$$

ii) where  $S_1 = (1, 2)$ ,  $S_2 = (1, 3)$ ,  $S_3 = (2, 3)$ . Show that:

a) Both  $\bar{y}$  and  $t$  are unbiased [8 marks]

b) Variance of  $\bar{y}$  is greater than variance of  $t$  and interpret the results. [6 marks]

c) Define the following terms as used in surveys:

i) Inclusion probability [2 marks]

ii) Sampling errors [2 marks]

iii) Non-sampling errors [2 marks]

- d) Signatures to a petition were collected on 676 sheets. Each sheet Had enough space for 42 signatures but on many sheets, a small number of signatures had been collected.

The numbers of signatures per sheet were counted on a random sample of 50 sheets with results tabulated below:

$y_i$	42	41	36	32	29	27	23	19	15	11
$f_i$	23	4	1	1	1	2	1	1	2	1
$y_i$	10	9	7	6	5	4	3	16	14	Total
$f_i$	1	1	1	3	2	1	1	2	1	=50

Estimate the following at 95% confidence limits:

- i) The total number of signatures. [3 marks]  
 ii) The mean of the total number of signatures. [3 marks]

### QUESTION TWO (20 MARKS)

- a) Show that  $\text{Var}(\bar{Y}_{\text{str}})_{\text{prop}} = \left(\frac{1-f}{n}\right) \sum_{i=1}^k W_i S_i^2$  [5 marks]

- b) The following data show the stratification of all the farms in a country by farm size and average acres of corn per farm in each stratum. For a sample of 100 farms, compute  $\text{var}(\bar{Y}_{\text{str}})$  under the following criterion. [7 marks]

- i) Proportional allocation  
 ii) Neyman allocation

Farm Size	Number of Farm, $N_i$	Average corn Acres, $\bar{Y}_i$	Standard Deviation $S_i$
0 - 40	394	5.4	8.3.
41 - 80	461	16.3	13.3
81 - 120	391	24.3	15.1
121 - 160	334	34.5	19.8

- c) Explain the advantages of survey sampling. [8 marks]

**QUESTION THREE (20 MARKS)**

- a) Completer the following table of a systematic sample each of size n. [4 marks]

Random start	Sample composition	Probability	Mean

- b) Explain the following terms:
- i) Target population [2 marks]
  - ii) Study population [2 marks]
  - iii) Population characteristic [2 marks]

- c) Consider the following model

$$E_{\mathcal{S}}(Y_i) = \beta x_i$$

$$COV_{\mathcal{S}}((Y_i, Y_j)_{x_i, x_j}) = \begin{cases} \sigma^2 x_i & i = j \\ 0 & i \neq j \end{cases}$$

Show that the ratio estimator of the finite population model

$$T = \sum_{i=1}^k y_i \text{ which is predicted using } y_i = \beta x_i + \varepsilon_i \text{ is the best linear unbiased estimator of T} \quad [10 \text{ marks}]$$

**QUESTION FOUR (20 MARKS)**

Discuss the principal steps of organizing a survey. [20 marks]