Question One (30 Marks)

a) Classify the differential equations below

   i) $2\frac{d^2y}{dx^2} - 3y\frac{dy}{dx} + y = 0$ (2 Marks)

   ii) $4xy\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = cos(2x)$ (2 Marks)

b) Set up a differential equation whose general solution is $y = cx^2 - 3x$ where c is an arbitrary constant   (4 Marks)

c) Find the general solution to the first order differential equations below

   i) $2xy^2dy - (y + xy)dx = 0$ (4 Marks)

   ii) $(6xy^3 - e^y)dx + (9x^2y^2 + 8y - xe^y)dy = 0$ (5 Marks)

d) Show that the solutions $x_1 = 3e^{7t}, \quad y_1 = 2e^{7t}$ and $x_2 = e^{-t}, \quad y_2 = -2e^{-t}$ of the system
   $\frac{dx}{dt} = 5x + 3y$
   $\frac{dy}{dt} = 4x + y$
   are linearly independent   (3 Marks)

e) Find the general solution to the second order differential equation
   $3\frac{d^2y}{dx^2} - 7\frac{dy}{dx} - 6y = 0$ (5 Marks)

f) The temperature x of an object at any time t is defined by the differential equation
   $\frac{dx}{dt} = -k(x - 30)$ where $k = 0.07$. Find the expression for the temperature of the object at any time t given that the object is initially heated to 200^0C (5 Marks)

Question Two (20 Marks)

a) Find the general solution to the differential equation

$\frac{dy}{dx} - 3y = e^{2x}$ (4 Marks)

b) Find the general solution to the differential equation $(x+2)dy + (y - e^x)dx = 0$ (5 Marks)

c) Find the general solution to the differential equation $(x - y)dx - (x + y)dy = 0$ (6 Marks)

d) A metal rod heated to 180^0c cools to 90^0c in 10 minutes when put in a room maintained at a constant temperature of 20^0c. Find the temperature of the metal rod at any later time t minutes. (5 Marks)

Question Three (20 Marks)

a) Find two linearly independent solutions of the differential equation $y'' + 5y' + 6y = cos(3x)$ hence find the Wronskian (7 Marks)

b) Find the general solution to the system of differential equations

$\frac{dx}{dt} = 6x - 3y$

$\frac{dy}{dt} = 2x + y$ (13 Marks)

Question Four (20 Marks)

a) A tank contains 20 kg of salt dissolved in water to form 100 litres of brine. Another salt solution containing 2.0 kg/litre of salt flows into the tank at a rate of 5 litres per minute and the mixture leaves the tank at the same rate. Calculate the amount of salt present in the tank at any later time t minutes. (8 Marks)

b) Use the method of variation of parameters to find the general solution to the differential equation $\frac{d^2y}{dx^2} - 4y = e^{5x}$ (12 Marks)

Question Five (20 Marks)

a) Find the general solution to the equi-dimensional equation

$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ (10 Marks)

b) Use the D-operator method to find the general solution to the differential equation

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 17y = cos(4x)$ (10 Marks)