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**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE**

**IN ACTUARIAL SCIENCE**

**3RD YEAR 1ST SEMESTER 2015/2016 ACADEMIC YEAR**

**REGULAR**

**COURSE CODE: SAS 303**

**COURSE TITLE: ESTIMATION THEORY**

**EXAM VENUE: STREAM: BSC. ACTUARIAL SCIENCE**

DATE: EXAM SESSION:

TIME: 2.00 HOURS

**Instructions:**

1. **Answer Question ONE and ANY other two questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

**QUESTION ONE (30 MARKS)**

1. Explain clearly the following terms as used in estimation theory.
2. Unbiasedness
3. Factorization criterion
4. Minimal sufficiency (6marks)
5. Let are iid random variables. By the factorization criterion, find a jointly sufficient statistic for the population mean and variance. (6 marks)
6. Suppose we have a random sample of size 2n from a population denoted by and , Var (X) = . Let , be two estimators of. Which between these two estimators is more efficient for estimation of (6marks)
7. Let be iid Bernoulli random variables with parameter

Obtain Fisher’s information for estimation of hence give the C.R.L.B for estimation of . (6marks)

1. Let , be a random sample of size n from Show that is biased for and state the amount of bias. (6marks)

**QUESTION TWO (20 MARKS)**

1. Let be iid random variables from the uniform distribution. Show that  
   is a consistent estimator of (13 marks)
2. Using the Lehmann Scheffe method, find the minimal sufficient statistic for given are iid random variables from (7marks)

**QUESTION THREE (20 MARKS)**

1. Let with . Let the set of actions with the loss function defined as

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  | 6 | 4 |
|  | 5 | 7 |

Suppose the class of all possible decisions is defined as.

Where ,,, ,

Determine for X the minimax rule. (8 marks)

1. You are provided with the following data

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 25 | 31 | 16 | 19 | 21 | 36 | 18 |
| 23 | 26 | 18 | 35 | 36 | 28 | 15 |

1. Calculate the point estimate for population mean and its associated estimate of population variance.
2. Calculate the point estimate for population median and its associated estimate of population variance.
3. Suppose one chooses the sample median over the sample mean to estimate the population mean, would this be a wise decision? (12 mks)

**QUESTION FOUR (20 MARKS)**

1. Let be a random sample from some population with finite mean and finite variance. Show that the sample variance is unbiased for the population variance. (8marks)
2. i. Explain the term weak consistency (3 marks)

ii. Let be iid random variables with and . Show that is a consistent estimator of (9marks)

**QUESTION FIVE (20 MARKS)**

1. Let X be gamma random variable with probability density function

Show that belongs to a 1- parameter exponential family. (5marks)

1. i. Explain what is meant by the term sufficiency (2 marks)
2. Let are iid random binomial (1,P) random variables. A loaded coin is tossed n times with probability of success as P. Show that to estimate P it is sufficient to know the statistic (6marks)
3. Suppose , is a random sample of size n from Obtain the UMVUE for , and based on the sample

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| height | 60-62 | 63-65 | 66-68 | 69-71 | 72-74 |
| frequency | 5 | 18 | 42 | 27 | 8 |

(7 marks)