Pwani UNIVERSITY

UNIVERSITY EXAMINATIONS 2013/2014 ACADEMIC YEAR

2nd YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE, BACHELOR SCIENCEBACHELOR OF ARTS

COURSE CODE/TITLE: SMA 261- PROBABILITY AND STATISTICS II

END OF SEMESTER: II DURATION: 3 HOURS

DAY/TIME: WEDNESDAY 8.00 TO 11.00AM DATE: 9.04.2014 (LTN)

Instructions:
Answer Question ONE and any other TWO questions.

SECTION A
QUESTION ONE

a) State the conditions discrete random variables X and Y must satisfy to have a joint probability density function (2marks)

b) Show that $E\left\{E(X/Y)\right\} = E(X)$ (4 marks)

c) Suppose that X and Y are discrete random variables with joint probability density function

$$f(x,y) = \begin{cases} p(2xy + x + y + 1) & \text{x} = 0,1,2 \quad , \text{y} = 1,3 \\ 0 & \text{otherwise} \end{cases}$$

I) Determine the value of P.

II) Determine the marginal density functions and
Hence check the independence of the random variable.

iii) Find the probability

a) $f(1,2)$

b) $f(X \le 1 \quad \text{Y} = 1)$ (10 marks)

d) Given that the var(x)=9 and var(y)=4 and $\ell_{xy} = \frac{-1}{6}$ compute variance of $[X - 3Y + 3]$ (4 marks)

e) Let Y be a continuous random variable. Show that the function

$$f(y) = \begin{cases} \frac{1}{2}y, & 0 \le y \le 2 \\ 0 & elsewhere \end{cases}$$

Is a probability density function. Hence calculate

$P(0.5 \le Y \le 1)$ and P(-I ≤ Y ≤ I) (6marks)

f) State the central limit theorem (3 marks)

g) State and prove the three properties of Bivariate conditional expectation (9 marks)

SECTION B

QUESTION TWO(15 marks)

a)) Consider the following joint p.d.f of two independent random variable

$$f(x,y)=\begin{cases}4xy & 0\le x\le 1 \;\; 0\le y\le 1\\ 0 & elsewhere\end{cases}$$

$$T = X + Y$$

$$Z = 2X - Y$$

Determine whether Z and T are independent

(9 marks)

b) Consider marks obtained by ten students in tests X and Y

student	A	B	C	D	E	F	G	H	I	J
X	13	30	15	9	17	22	8	17	14	12
Y	21	12	19	22	18	13	25	15	20	16

Compute the rank correlation coefficient (6 marks)

QUESTION THREE(15 marks)

a)Suppose that X and Y are two discrete random variables with the joint p.d.f

$$f(x,y)=\begin{cases}a(xy+x+\tfrac{1}{2}y+1) & x=0,1; y=2,4\\ 0 & elsewhere\end{cases}$$

(i) Determine value of a (2 marks)

(ii) Determine the joint moment generating function (3marks)

(iii) Determine the mean of x and that of y (4 marks)

(iv) Determine the E(XY) hence check for independence (3 marks)

(v) What is the correlation coefficient (3 marks)

QUESTION FOUR(15 marks)

a)Show that var $(ax-by)=a^2\,\text{var}(x)+b^2\,\text{var}(y)-2ab\delta xy$ (4marks)

b) Using the moment generating technique show that the mean and the variance of the χ^2 distribution are n and 2n given that (11 marks)

$$f(x)=\begin{cases}\dfrac{1}{\Gamma\left(\frac{n}{2}\right)2^{\frac{n}{2}}}x^{\frac{n}{2}-1}e^{\frac{-x}{2}} & x>0\\ 0 & elsewhere\end{cases}$$