Pwani University College

A Constituent College of Kenyatta University

UNIVERSITY EXAMINATIONS 2011/2012 ACADEMIC YEAR

2^{ND} YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF

BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION SCIENCE

COURSE CODE/TITLE: SMA 230 – VECTOR ANALYSIS

END OF SEMESTER: I DURATION: 3 HOURS

DAY/TIME: MONDAY 8.00 TO 11.00AM DATE:28.11.2011 (GS1)

Attempt question ***ONE*** in Section A and any ***TWO*** questions from section B

SECTION A

Question One (40 marks)

a) Prove that if vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are not parallel and $\lambda\underset{\sim}{a}+\beta\underset{\sim}{b}=\underset{\sim}{0}$, then $\lambda=\beta=0$.

(4 Marks)

b) Find the question of the place through $A(1,2,3)$, $B(4,2,5)$ and $C(5,4,6)$.

(3 marks)

c) Show the relationship between the plane $2x+y-z=8$ and the line whose equation is $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$

(4 Marks)

d) Find the angle between the plane $2x+y-2z=5$ and the line $\frac{x-2}{1}=\frac{y-1}{2}=\frac{z+1}{4}$

(4 Marks)

e) Show that the area of a parallelogram is equal to the cross product of the vectors defining the parallelogram. (5 Marks)

f) Show that if $B(t)$ has a constant magnitude, then $\frac{dB}{dt}$ is perpendicular to $\underset{\sim}{B}$, (5 Marks)

g) Show that $\frac{d}{dt}\left(v.\frac{dv}{dt}\times\frac{d^2v}{dt^2}\right)=v.\frac{dv}{dt}\times\frac{d^3v}{dt^3}$ (3 Marks)

h) A curve has a parametric equation $x=3\cos t$, $y=3\sin t$, $z=4t$, find

i). The unit tangent vector $\underset{\sim}{T}$ (3 Marks)

ii). The principle normal, $\underset{\sim}{N}$, the curvature k and radius of curvature ρ. (4 Marks)

iii). The binormal $\underset{\sim}{B}$, torsion τ and radius of torsion δ. (5 Marks)

SECTION B

Question Two (15 marks)

a) Find the directional derivative of $\phi=x^2yz+4xz^2$ at $(1,-2,-1)$ in the direction of the vector $2i-j-2k$. (5 Marks)

b) Given $\underset{\sim}{A}=xz^3i+2x^3yzj+2yz^4k$, evaluate the curl of *A*. (3 Marks)

c) i). Find the constant *a*, *b*, and *c* for $\underset{\sim}{V}=(x+2y+az)i+(bx-3y-2)j+(4x+cy+2z)k$ to be irrotational

ii). Find the scalar potential (7 Marks)

Question Three (15 marks)

a) Show that the diagonals of *a* parallelogram bisect each other. (5 Marks)

b) Prove the sine rule (5 Marks)

c) Prove that for any scalar function ϕ, the curl grad ϕ is zero. (5 Marks)

Question Four (15 marks)

a) State the Green's Theorem on the plane (5 Marks)

b) Verify Green's theorem for $\oint_c \left(xy + y^2\right)dx + x^2 dy$ where c is the closed curve bounded by $y = x$ and $y = x^2$ (10 Marks)