A Constituent College of Kenyatta University

UNIVERSITY EXAMINATIONS 2012/2013 ACADEMIC YEAR

2nd YEAR EXAMINATION FOR THE DEGREE OF BACHELOR SCIENCE AND BACHELOR OF EDUCATION SCIENCE

COURSE CODE/TITLE: SMA 202 – LINEAR ALGEBRA I

END OF SEMESTER: I **DURATION: 3 HOURS**

DAY/TIME: MONDAY 2.00 TO 5.00PM DATE: 3.12.2012 (PL1/PL2)

Instructions: Answer question ONE in section A and any other two in section B

SECTION A

QUESTION ONE –COMPULSORY (40 MARKS)

a. Define the determinant of a general 3×3 matrix. (4 marks)

b. Define (i) a diagonal matrix
(ii) an upper triangular matrix
(iii) a singular matrix (9 marks)

c. Let the vectors $\vec{a} = (1,2,3)$ and $\vec{b} = (2,0,1)$.

Determine (i) $\vec{a}.\vec{b}$

(ii) $\vec{a}\times\vec{b}$

(iii) the obtuse angle between $\vec{a}$ and $\vec{b}$ (8 marks)

d. Matrices P and Q are members of a set R which is defined as follows,

$R = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \Re, ad - bc = 1 \right\}$. Show that the product of P and Q is also a member of set R. (10 marks)

e. Let $A = \begin{pmatrix} 2 & -3 & -4 \\ -1 & 4 & 2 \\ 3 & 10 & 1 \end{pmatrix}$, compute (i) adjA

(ii) adjA(A) and A(adjA)

(iii) detA

(iv) derive a formular for the inverse of A using adjA

(9 marks)

SECTION B

QUESTION TWO-15 MARKS

a. By the Gauss Jordan Elimination method, solve the system

$$x + 2y + z = 4$$
$$3x + 8y + 7z = 20$$
$$2x + 7y + 9z = 23$$

(7 marks)

b. Derive the formular for the distance from the origin to the plane $ax + by + cz = d$

(8 marks)

QUESTION THREE-15 MARKS

a. Define linear dependence and independence of vectors in $\Re^n$ (6 marks)

b. Investigate whether or not the following vectors are linearly dependent.

$v_1 = (1, -2, 4, 1), v_2 = (2, 1, 0, -3)$ and $v_3 = (3, -6, 1, 4)$ (5 marks)

c. Show that the matrices, $A = \begin{pmatrix} 1 & 1 \\ 4 & 4 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ and C= $\begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix}$ are linearly dependent. (4 marks)

QUESTION FOUR-15 MARKS

a. Define a subspace W of a vector space V (3 marks)

b. Let $u_1, u_2, ..., u_n \in \Re^n$ be vectors and W be the set of all the linear combinations of $u_1, u_2, ..., u_n \in \Re^n$. Show that W is a subspace of $\Re^n$. (5 marks)

c. Let W be a subspace of $\Re^4$ generated by the vectors (1,-2,5,-3), (2,3,1,-4) and (3,8,-3,-5). Find (i) a basis and dimension of W

(ii) extend the basis of W to a basis of vectors in $\Re^4$ (7 marks)