UNIVERSITY EXAMINATIONS 2013/2014 ACADEMIC YEAR

2nd YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION SCEINCE, BACHELOR OF EDUCATION ARTS, BA

COURSE CODE/TITLE: SMA 202 EXAMINATION- LINEAR ALGEBRA I

END OF SEMESTER: I **DURATION: 3 HOURS**

DAY/TIME: THURSDAY: 8.00 - 11.00AM **DATE: 4/12/2013 (S3)**

Instructions: Answer question ONE in Section A and any other TWO questions in Section B.

Section A

QUESTION ONE – (40 MARKS)

a. Define a low reduced echelon matrix (5 marks)

b. Reduce $\begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 1 & 2 & 0 \\ -1 & -2 & -1 & 1 \\ 1 & -1 & 3 & 3 \end{pmatrix}$ to row reduced echelon form

(5 marks)

c. Determine the solution of the homogenous system $\begin{matrix} a_1x+b_1y+c_1=0 \\ a_2x+b_2y+c_2=0 \end{matrix}$ expressing your answer in determinant form (7 marks)

d. A system of linear equations may have a unique solution, many solutions or no solution. Explain how you would know whether a particular system has any of the above solutions

(4 marks)

e. Explain how you would use the cramer's rule to determine the solution to a system of equations in n variables (6 marks)

f. Determine the distance between the planes $x+2y-2z=3$ and 2x+4y-4z=7 (4 marks)

g. Determine whether the given vectors in $\Re^4$ are linearly dependent or not $(1,3,-1,4),(3,8,-5,7),(2,9,4,23)$ (4 marks)

h. Define a vector space and state two examples of vector spaces (5 marks)

Section B

QUESTION TWO – (15 MARKS)

a. Let $A=\begin{pmatrix} 2 & -4 & 6 \\ 3 & -6 & 1 \\ -2 & 5 & -2 \end{pmatrix}$. Use the method of the adjoint to find the inverse of A. hence solve the system

$$\begin{aligned} 2x-4y+6z&=20 \\ 3x-6y+z&=22 \\ -2x+5y-2z&=-18 \end{aligned}$$

(7 marks)

b. Let vectors $u_1,u_2,..........,u_m \in \Re^n$ and W be the set of all linear combinations of $u_1,u_2,..........,u_m \in \Re^n$. Prove that W is a subspace of $\Re^n$ (3 marks)

c. Define a basis of a finite subspace of a vector space V. Determine whether or not $\{(1,1),(2,1)\}$ is a basis of $\Re^2$ (5 marks)

QUESTION THREE – (15 MARKS)

a. Show that the lines $r=(3,5,7)+m(1,2,1)$ and s=$(1,2,3)+n(2,3,5)$ do not meet. (5 marks)

b. Write the polynomial $v=t^2+4t-3$ as a linear combination of the polynomials $e_1=t^2-2t+5, e_2=2t^2-3t$ and $e_3=t+3$ (6 marks)

c. Find the line of intersection of the planes $3x+2y-4z-6=0$ and x-3y-2z-4=0 (4 marks)

QUESTION FOUR – (15 MARKS)

a. Show that for any two matrices A and B, $(A-B)(A+B)=A^2-B^2$ if and only if A and B commute. (5 marks)

b. Let $x=x_1i+x_2j+x_3k, y=y_1i+y_2j+y_3k$ and z=$z_1i+z_2j+z_3k$ be vectors. Show that

$$x.(y\times z)=\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$$

(5 marks)

c. Determine the equation of the plane through the points $A(1,1,1), B(2,0,2)$ and $C(-1,1,2)$ (5 marks)