



MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

P.O. Box 972-60200 – Meru-Kenya.

Tel: 020-2069349, 061-2309217. 064-30320 Cell phone: +254 712524293, +254 789151411

Fax: 064-30321

Website: www.must.ac.ke Email: info@must.ac.ke

University Examinations 2013/2014

THIRD YEAR, SECOND SEMESTER EXAMINATIONS FOR DEGREE OF BACHELOR OF
COMPUTER SCIENCE

SMA 2305: COMPLEX ANALYSIS I

DATE: DECEMBER 2013

TIME: 2 HOURS

INSTRUCTIONS: Answer question *one* and any other *two* questions

QUESTION ONE – (30 MARKS)

- (a) Work out and write your answer in the form $x + iy$

$$\frac{1+2i}{3-4i} + \frac{2-i}{5i} \quad (4 \text{ Marks})$$

- (b) (i) State Cauchy's integral formula. (1 Mark)

(ii) Use Cauchy's integral formula to evaluate

$$\int_c \frac{\cos(z)}{z^3 + z} dz \quad \text{where } c \text{ is } |z| = \frac{1}{2} \quad (4 \text{ Marks})$$

- (c) Show that for any two complex numbers $z_1 = x_1 + iy$, and $z_2 = x_2 + iy_2$ then
 $\overline{z_1 z_2} = \overline{z_1} \overline{z_2}$ (5 Marks)

- (d) Find the real and the imaginary part of the function. $f(z) = \frac{z^2 + z}{|z|^2}$ (4 Marks)

- (e) Show that $u(x, y)$ is harmonic in some domain and finds its harmonic conjugate $v(x, y)$
given that $u(x, y) = 2x - x^3 + 3xy^2$. (5 Marks)

(f) Determine the Residue of the function $f(z) = \frac{e^z}{z^2 + 2z - 3}$ at each of its poles in finite z -plane. (3 Marks)

(g) Find all the solutions to the equation $z^4 + 4 = 0$ using Demoiver's theorem (4 Marks)

QUESTION TWO – (20 MARKS)

(a) Evaluate the contour integral of $f(z) = z^2$ ie $\int_c z^2 dz$ where c_1 is the line segment from $-1 - i$ to $3 + i$ and c_2 is the portion of the parabola $x = y^2 + 2y$ joining $-1 - i$ to $3 + i$ (12 Marks)

(b) Using Cauchy's integral formula evaluate $\oint_c \frac{5z dz}{(z+1)(z-2)(z-4i)}$ if

(i) $c: |z| = 3$ (4 Marks)

(ii) $c: |z| = 5$ (4 Marks)

QUESTION THREE – (20 MARKS)

(a) If z_1 and z_2 are arbitrary complex numbers on a complex plane. Show that

$$|z_1 + z_2| \leq |z_1| + |z_2| \quad (6 \text{ Marks})$$

(b) Test whether the following functions are analytic

(i) $f(z) = 3x + y + i(3y - x)$ (3 Marks)

(ii) $f(z) = e^{-y} \sin x - i e^{-y} \cos x$ (4 Marks)

(c) If $f(z) = u(x, y) + iv(x, y)$, show that u and v are harmonic in region R . If and only if the function $f(z)$ is analytic in Region R . (7 Marks)

QUESTION FOUR – (20 MARKS)

(a) Define a residue of a function $f(z)$ at a simple pole $z = z_0$. (1 Mark)

(b) Determine the residue of the function $f(z) = \frac{2z}{(z^2 + 1)(2z - 1)}$ at each of its poles in the finite Z -plane. (8 Marks)

(c) Evaluate the contour integral $\oint_c \frac{dz}{z^3(z^2 + 2z - 3)}$ using Cauchy's residue theorem where c is

(i) $|z| = \frac{1}{2}$ (6 Marks)

(ii) $|z| = 2$ (5 Marks)