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University Examinations 2012/2013

THIRD YEAR, FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE, BACHELOR OF SCIENCE IN STATISTICS AND BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

SMA 2301/STA 2306: REAL ANALYSIS 1/REAL ANALYSIS FOR STATISTICS

DATE: AUGUST 2013 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Let A and B be sets. Define the following terms as used in sets
   i. $A \underline{C} B$ (2 Marks)
   ii. $A \cap B$ (2 Marks)
   iii. $A \cup B$ (2 Marks)
   iv. $A|B$ (2 Marks)

b) Prove that the square of an odd number is odd. (3 Marks)

c) Let $X \in \mathbb{R}^2$ and define $d: \mathbb{R}^2 \to \mathbb{R}^2$ by
$d\big((x_1, y_1)(x_2, y_2)\big) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ for all points (x_1, y_1) and (x_2, y_2) in $\mathbb{R}^2$. Show that d is a metric on $\mathbb{R}^2$. (4 Marks)

d) Let $x_n = \left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, -\frac{2}{3}, \frac{1}{4}, \frac{3}{4}, -\frac{3}{4}, \frac{1}{5}, \frac{4}{5}, -\frac{4}{5}, \ldots.\right)$. find the set S of sub sequential limits, limit superior and limit inferior.(4 Marks)

e) Show that $f(x) = 3X - 5$ is uniformly continuous on $\{-1,1\}$. (3 Marks)

f) Let $a_n = \begin{cases} \frac{n}{2^n}, if\ n\ is\ odd \\ \frac{1}{2^n}, if\ n\ is\ even \end{cases}$

Determine whether $\sum a_n$ converges. (4 Marks)

g) Determine the interior of the sets:
   i. $[7, 10]$ (2 Marks)
   ii. $T = \{a\sqrt{2}, a \in \mathbb{N}\}$. (2 Marks)

QUESTION TWO (20 MARKS)

a) i) Define an open set. (3 Marks)

ii) Prove that the intersection of finite collection of open sets is open. (5 Marks)

b) Use the set $A_n = \left(2 - \frac{1}{n}, 3 + \frac{1}{n}\right) : \forall n \in \mathbb{N}$ to show that the intersection of an arbitrary collection of open sets is not necessarily open. (4 Marks)

c) i) Define a limit point of a subset of $\mathbb{R}$. (3 Marks)

ii) Prove that finite sets have no limit points. (5 Marks)

QUESTION THREE (20 MARKS)

a) i) Define the term convergence of sequences. (3 Marks)

ii) Show from first principles that $\lim_{n\to\infty}\left(\frac{3n+2}{n+1}\right) = 3$. (4 Marks)

b) Prove that a sequence (x_n) in $\mathbb{R}$ has a unique limit. (5 Marks)

c) Use the sequence $(x_n) = (-)^n\left(1 + \frac{1}{n}\right) \forall n \in \mathbb{N}$, to show that boundedness does not imply convergence. (4 Marks)

d) Compute $\lim_{n\to\infty}\left(\frac{4n^2-3}{5n^2-2n}\right)$ (4 Marks)

QUESTION FOUR (20 MARKS)

a) Define the following terms as used in series:

i. Absolute convergence. (2 Marks)

ii. Conditional convergence. (2 Marks)

b) Determine whether $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$ is conditionally convergent. (4 Marks)

c) i) Define continuity of a function. (3 Marks)

ii) Show that the function $f(x) = \begin{cases} \frac{x^2-1}{x-1}, x \neq 1 \\ 2, \ x = 1 \end{cases}$ is continuous at x = 1 (3 Marks)

d) i) Show that the function $f(x) = \begin{cases} \frac{2x^2-3x+1}{x-1}, x \neq 1 \\ 5, \ x = 1 \end{cases}$ is discontinuous at x = 1.

(3 Marks)

ii) Let $f(x) = \frac{x^2-4x-5}{x-5}$, for $x \neq 5$. How should f(x) be defined so that f is continuous at $x = 5$? (3 Marks)