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**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL**

**1ST YEAR 1ST SEMESTER 2017/2018 ACADEMIC YEAR**

**REGULAR (MAIN)**

**COURSE CODE: SAC 103**

**COURSE TITLE: MATHEMATICAL MODELLING**

**EXAM VENUE: STREAM: (Bsc. ACTUARIAL SCIENCE)**

**DATE: EXAM SESSION:**

**TIME: 2.00 HOURS**

**Instructions:**

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

**QUESTION ONE COMPULSORY (30 MARKS)**

1. Define the following terms (8 Marks)
2. Difference equation
3. Differential equation
4. Homogeneous difference equation
5. The order of difference equation
6. It is known that for certain income brackets, money deposited in an individual account retirement account (IRA) and its interest are tax-deferred. Karen Smith sets up an IRA at the beginning of the year and deposited $3,000. Karen will deposit $2,000 at the end of the year, and the interest rate is 6% compounded annually.
7. Model this situation by a difference equation. (3 Marks)
8. What will be the future value of the account at Karen’s retirement age of 60 if she starts the IRA at the age of 25? (5 Marks)
9. A drug is administered to a patient at a constant rate c. As it is administered through it is converted to the patient’s body to other substances at the rate proportional to its current concentration. Formulate a differential equation for the concentration of the drug, D, changes with time. (3 Marks)
10. Find the interest on a 60-day loan of $1500 at 4.15%. (5 marks)
11. Solve the following difference equations to find 
12. ,   (3 Marks)
13. ,  (3 Marks)

**QUESTION TWO (20 MARKS)**

1. Assume that you open an account that pays 4% compounded annually and deposit $2,000. You will deposit 2% more into your account than you deposited in the previous year.
2. Model this situation with a difference equation.
3. What is the total amount in the account after 20 years?
4. You just won the lottery. You put your $5,000,000 in winnings into a fund that has a rate of return of 4%. Each year you use $300,000. How much money will you have 20 years from now?

**QUESTION THREE (20 MARKS)**

1. You have a balance of $5,000 on a credit card which charges 2% interest per month. You promise to pay dollars a month to the credit card company and not make any new charges.
2. Formulate a model in terms of which allows you to pay a credit card in 10 years.
3. Solve your model analytically to find the value (to the nearest cent) which will allow you to pay off the credit card in exactly 10 years. (10 Marks)
4. Suppose it is known that the cells of a given bacterial culture divide every 3.5hrs on average. If there are 500 cells in dish to begin with, how many will there be after 12 hours?

**QUESTION FOUR (20 MARKS)**

1. Your patients are considering a 30 year, $100,000 mortgage that charges 0.5% interest each month. They want to pay  monthly so that the mortgage (loan) is paid off after 360 payments (i.e. 30 years).
2. Set and solve the model with the dynamical system (i.e. difference equation)
3. Find the monthly payment  (10 Marks)
4. Consider the following predator-prey model for two species and. Assume that when the predator and the prey meet, it is good for the population of the predator and bad for the prey.





1. Identify which variable represent the predator and which one represent the prey. Explain your answer.
2. According to the model, what happens to the population of the predator if there are no preys present?

**QUESTION FIVE (20 MARKS)**

 Suppose the population of a country is split into two groups:

 Group 1: Consisting on 0-12 years olds and

 Group 2: Consisting of the rest.

 Assume that births only occur in group 2. Each group will only have its own death rate.

 Define as the population of the 0-12 group in year.

 as the population of the 13+ group in year .

  as the birth rate

 as the death rate in the 0-12 group

 as death rate in the 13+ group

 Assume that further the survivors of group 1 transfer to group 2 each year.

Required: Form difference equations for the two groups. Let million,  million and the population parameters are;

 ,  and 0.03

Required: Calculate the population after one year.