



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN
ACTUARIAL SCIENCE**

2ND YEAR 1ST SEMESTER 2017/2018 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SAC 201

COURSE TITLE: FINANCIAL MATHEMATICS I

EXAM VENUE:

STREAM: (BSc. Actuarial)

DATE:

EXAM SESSION:

TIME: 2.00 HOURS

Instructions:

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION 1 [COMPULSORY] [30 Marks]

(a) Prove that

$$S_{\overline{m+n}|} = (1 + i)^n S_{\overline{m}|} + S_{\overline{n}|}$$

[3 Marks]

(b) The rate of interest is 4.5% per annum effective.

(a) Calculate:

- (i) the annual effective rate of discount.
- (ii) the nominal rate of discount per annum convertible monthly.
- (iii) the nominal rate of interest per annum convertible quarterly.
- (iv) the effective rate of interest over a five year period.

[8 Marks]

(c) You are offered a loan of Kshs 10,000 with no payments for 6 months, the Kshs.600 per month for 1 year, and Kshs.500 per month for the following year. What annual effective rate of interest does this loan charge?

[5 Marks]

(d) The force of interest $\delta(t)$ is a function of time, and at any time t , measured in years, is given by

$$\delta(t) = 0.03 - 0.005t + 0.001t^2 \qquad 0 \leq t \leq 10$$

(i) Calculate the equivalent constant force of interest per annum for the period $t = 0$ to $t = 10$ **[4 Marks]**

(ii) Calculate the accumulated value at time $t = 7$ of an investment of

Kshs.250,000 at time $t = 0$ plus a further investment of Kshs.150,000 at time $t = 5$ [4 Marks]

(e) A man borrows Kshs.750,000 to buy a car. He repays the loan by 24 monthly installments in arrears. The flat rate of interest is 9% p.a

(i) What is his monthly repayment? [3 Marks]

(ii) What is the APR on this transaction? [3 Marks]

QUESTION 2 [20 Marks]

(a) Define the accumulation factor $A(t, t + h)$ and give a formula for the force of interest $\delta(t)$ per unit time in terms of the accumulation factor.

[4 Marks]

(b) The force of interest $\delta(t)$ at time t (measured in years) is given by $\delta(t) = 0.001t + 0.04$

(i) Calculate the corresponding nominal rate of interest for the period $t = 1$ to $t = 2$ [5 Marks]

(ii) If an investment of 1 is made at time $t = \frac{1}{2}$ calculate the value to which it will have accumulated at time $t = 6$ [5 Marks]

(c) Calculate the accumulated value of an investment of Kshs.100 for 6 months at the following rates of interest

(i) a force of interest of 0.05 p.a [2 Marks]

(ii) a rate of interest of 5% p.a convertible monthly [2 Marks]

(iii) an effective rate of interest of 5% p.a [2 Marks]

QUESTION 3 [20 Marks]

The force of interest per unit time at time t , $\delta(t)$, is given by:

$$\delta(t) = \begin{cases} 0.1 - 0.005t & t < 6 \\ 0.07 & 5t \geq 6 \end{cases}$$

(a) Calculate the total accumulation at time 10 of an investment of Kshs.100,000 made at time 0 and a further investment of Kshs.50,000 made at time 7. **[10 Marks]**

(b) Calculate the present value at time 0 of a continuous payment stream at the rate Kshs.50,000 $e^{0.05t}$ per unit time received between time 12 and time 15. **[10 Marks]**

QUESTION 4 [20 Marks]

A bank makes a loan to be repaid in installments annually in arrears. The first installment is 500, the second 480 and so on with the payments reducing by 20 per annum until the end of the 15th year after which there are no further payments. The rate of interest charged by the lender is 6% per annum effective.

(a) Calculate the amount of the loan **[10 Marks]**

(b) Calculate the interest and the capital components of the second pay-

ment [5 Marks]

(c) Calculate the amount of capital repaid in the installment at the end of the fourteenth year [5 Marks]

QUESTION 5 [20 Marks]

A company is set to refurbish old mills and turn them into retail outlets. The company purchases 5 mills each costing Kshs.500,000. One mill is purchased at the start of each of the next 5 years. The cost of refurbishment is Kshs.20,000 per mill and is payable continuously for one year after the purchase. Once the refurbishment for a particular mill is complete, retail stores pay rent to the company for the mill at a rate of Kshs.48,000 per year payable monthly in arrears. Each mill is sold 10 years after completion of its refurbishment for Kshs. 600,000.

The company employs a manager to run this project. She is paid Kshs.50,000 p.a payable at the end of each month whilst the company has ownership of any of the mills.

Calculate

- (a) The net present value of the project assuming an interest rate of 4% p.a effective [12 Marks]
- (b) The discounted payback period [5 Marks].
- (c) The accumulated profit on the day the last mill is sold [3 Marks]