JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

3RD YEAR 1ST SEMESTER 2016/2017 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SAC 303

COURSE TITLE: ACTUARIAL MATHEMATICS II

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

(a) Define the following terms as used in actuarial mathematics;

i. Reversionary annuities.

ii. Contingent probability.

iii. Life annuities.

iv. ${}_tq_{xy}$.

v. ${}_tp_{\overline{xy}}$. [5 marks]

(b) For two independent lives now age 30 and 34, you are given:

x	q_x
30	0.1
31	0.2
32	0.3
33	0.4
34	0.5
35	0.6
36	0.7
37	0.8

Calculate the probability that the last death of these two lives will occur during the 3^{rd} year from now (i.e. ${}_{2|}q_{\overline{30:34}}$). [5 marks]

(c) A continuous two-life annuity pays

i. 100 while both (30) and (40) are alive,

ii. 70 while (30) is alive but (40) is dead, and

iii. 50 while (40) is alive but (30) is dead.

The actuarial present value of this annuity is 1180. Continuous single life annuities paying 100 per year are available for (30) and (40) with actuarial present values of 1,200 and 1,000, respectively.

Calculate the actuarial present value of a two-life continuous annuity that pays 100 while at least one of them is alive. [7 marks]

(d) You are given:

i. ${}_{10|}q_{\overline{50:60}} = 0.0105$

ii. ${}_{10}p_{50} = 0.800$

iii. ${}_{10}p_{60} = 0.750$

iv. $p_{60} = 0.975$

Determine q_{70}. [5 marks]

(e) You are given two lives (x) and (y) such that

i. (x) is subject to a uniform distribution of deaths over each year of age.

ii. (y) is subject to a constant force of mortality of 0.3.

iii. $q^1_{xy} = 0.045$

iv. T_x and T_y are independent.

Calculate q_x. [3 marks]

(f) Calculate the probability that an active member, who is currently aged exactly 39, will retire in normal health over the year of age ending on his 62^{nd} birthday. Use service table defined in the Pension Scheme Tables. [2 marks]

(g) Write down an expression , using commutation functions, for the EPV of a lumpsum of \$ 150,000 paid immediately on the event of ill-health retirement for an active pension scheme member aged exactly 48. Assume that ill-health retirement is only permitted before the member reaches his 60^{th} birthday. [3 marks]

QUESTION TWO

(a) Assume that for two independent lives (50) and (60), mortality is described by

$$\mu_z = \frac{1}{100 - z}, \qquad \text{for } 0 \leq \ z < 100$$

Calculate $\mathring{e}_{\overline{50:60}}$ and interpret this value. [5 marks]

(b) Calculate the EPV of a lumpsum benefit of % 50,000 paid on normal age retirement, for a scheme member aged exactly 52, assuming that this benefit is paid;

i. regardless of the actual age at retirement.

ii. only if the member retires after the 64^{th} birthday. [5 marks]

(c) Jim and Dot, both aged 60, buy an annuity payable monthly in advance for atmost 20 years which is payable while atleast one of them is alive. Calculate the expected present value of the annuity assuming a 4% p.a interest. [10 marks]

QUESTION THREE

(a) Given that

$$\mu_x = \frac{1}{100 - x}, \ 0 \leq \ x < 100$$

. Calculate the value of ${}_{30}q_{60:50}{}^{2}$. [5 marks]

(b) For a temporary life annuity-immediate on two independent lives, you are given

i. Mortality follows an Illustrative Life Table and (ii) $i = 6\%$.

Calculate $e_{30:40:\overline{10|}}$. [3 marks]

(c) A pension scheme provides the following benefit to a spouse member following the death of a member in retirement.

A pension of \$ 10,000 p.a payable during the lifetime of the spouse, but ceasing 15 years after the death of the member if that is earlier. All payments are made on the anniversary of the member's retirement.

Calculate the expected present value of the spouse's benefit in the case of a female member retiring now on her 60^{th} birthday, who has a husband aged exactly 55 years. Basis: PA92620 mortality 4% interest. [7 marks]

(d) The table below shows the independent rates of ill-health retirements, withdrawals and deaths for a pension scheme for ages 20 and 40. Calculate the dependent rates of decrement at these ages assuming that each decrement is uniform over each year of age in its single decrement tables.

Age	ill-health retirement	Withdrawals	Death
20	-	0.25	0.002
40	0.01	0.05	0.003

[5 marks]

QUESTION FOUR

(a) For two independent lives (x) and (y), you are given:

i. T_x is subject to a constant force of mortality of 0.045.

ii. T_y is subject to a constant force of mortality of 0.035.

iii. Force of interest is $\delta = 5.0\%$.

Calculate $\bar{A}_{xy}$. [5 marks]

(b) For the future lifetimes of (50) and (80),

i. Deaths occur simultaneously (i.e $T_{50} = T_{80}$) with a probability of 0.3

ii. Otherwise with a probability of 0.7, the joint density function is

$$f_{T_{50};T_{80}}(s;t) = 0.001 \quad \text{for} 0 < s < 50,\ 0 < t < 20$$

Calculate the probability that (80) will outlive (50). [5 marks]

(c) A population is subject to two modes of decrement, α and β, in the single decrement tables;

$$_tp_{60}^{\alpha} = 1 - \frac{t}{40},\ 0 \leq\ t \leq\ 40$$

$$_tp_{60}^{\beta} = (1 - \frac{t}{40})^2,\ 0 \leq\ t \leq\ 40$$

Calculate the value of $(aq)_{60}^{\alpha}$ [6 marks]

(d) In a certain population, lives aged 40 are subject to three decrements- ill-health retirement, withdrawals and death. Each decrement operates uniformly over the year of age (40, 41) in the multiple decrement table. You are given $(aq)_{40}^{d} = 0.00291$, $(aq)_{40}^{w} = 0.04968$. Calculate the underlying independent rates. [4 marks]

QUESTION FIVE

(a) Calculate the probability of the following events using the service table;

i. A member aged exactly 40 will die from service during the next year. [1 mark]

ii. A member aged 38 exactly will retire through ill-health in the year ending on his 60^{th} birthday. [1 mark]

(b) You are given:

i. Male mortality follows De Moivres law with $\omega = 90$.

ii. Female mortality also follows De Moivres law where at age 80, the force of mortality is half that of the male force of mortality.

For two independent lives, a male age 75 and a female age 80, determine the expected time until the second death. [6 marks]

(c) The in-force expected cashflows for a five year unit-linked policy under which no non-unit reserves are held is

$$(-60.2,\ -2.5,\ -17,\ 50.13,\ 85.75)$$

Calculate the reserves required if negative cashflows other than in year one are to be eliminated and give the revised profit vector allowing for reserves. Assume that reserves earn an interest at a rate of 5% p.a. Ignore mortality. [7 marks]

(d) For a special fully continuous last survivor insurance of 1 on two independent lives (x) and (y), you are given

i. Death benefits are payable at the moment of second death.

ii. Level benefit premiums, π are payable only when $(x)$is alive and (y) is dead; no premiums are payable while both are alive or if (x) dies first.

iii. $\delta = 0.5,\ \mu_x(t) = 0.03,\ t \geq\ 0,\ \mu_y(t) = 0.04,\ t \geq\ 0$

Calculate 1000π. [5 marks]