c) i) Find the minimum polynomial for the matrix $A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{pmatrix}$ (3mks)

ii) State the algebraic multiplicity of the eigenvalues of A (2mks)

d) Let V be a linear space of all functions of the form $f(t) = C_1 cost - C_2 sint$ where C_1 *and* C_2 are arbitrary constants .

i) Find the matrix of the linear transformation $T: V \rightarrow V$ given by $T(f) = f' + af - bf''$ with respect to the basis $cost,\ sint$ where a and b are constants (3mks)

ii) Give the condition for the isomorphism of T (1mk)

iii) Find the function f *in* V such that $T(f) = f' + af - bf'' = \cos t$ (3mks)

ii) Find the non singular matrix P such that $D = P^{-1}AP$, where D is a diagonal matrix (3mks)

iii) Show that A satisfies Cayley Hamilton's theorem (2mks)

c) Let $S = \left\{\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}\right\}$ be the basis for $\mathbb{R}^2$. Find the coordinate vector of $\begin{pmatrix} 12 \\ 13 \end{pmatrix}$ with respect to S (3mks)

QUESTION FOUR (20 MARKS)

a) Show that the coordinate mapping is linear (3mks)

b) i) Find a basis of the space V of all polynomials $f(t)$ in P_4 such that $f'''(1) = 0$ (3mks)

ii) Determine the coordinate transformation of $\mathbb{R}^{2\times 2}$ with respect to the standard basis (3mks)

iii) Let T be a linear transformation defined by $T(f) = f'' + f'''$. Find the matrix of a linear mapping T with respect to the standard bases for P_4 (4mks)

c) Let $T : P_3 \rightarrow \mathbb{R}^{2\times 2}$ be a linear mapping given by $T(f) = f(A)$, where

$$A = \begin{bmatrix} -2 & 2 \\ 1 & 3 \end{bmatrix}$$

i) Find the matrix of T with respect to the standard basis $\beta = \{1, t, t^2, t^3\}$ (2mks)

ii) Use the matrix to find the bases for the image and kernel of T (5mks)

QUESTION FIVE (20 MARKS)

a) Find the matrix corresponding to the change of basis from $\beta = \left\{\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \end{bmatrix}\right\}$ $\alpha = \left\{\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix}\right\}$ (3mks)

b) i) Find the symmetric matrix representation of the quadratic form $q(x_1, x_2) = 2{x_1}^2 + 2x_1x_2 + 2{x_2}^2$ (4mks)

ii) Find the variables that reduce the quadratic form to a sum of squares (1mk)

ii) Let $T: P_3 \rightarrow P_2$ be given by $T(f) = f' + f''$. Find the matrix of a linear mapping T with respect to the standard basis $\beta = \{1,\ t,\ t^2,\ t^3\}$ (4mks)

iii) Diagonalize the matrix $A = \begin{bmatrix} 1 & 4 \\ 1 & -2 \end{bmatrix}$ (6mks)

e) Let $T: P_2 \rightarrow \mathbb{R}^{4\times 3}$ be a linear mapping given by $T(f) = f(A)$, where $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.

i) Find the matrix of T with respect to the standard basis $\beta = \{1, t, t^2\}$ (2mks)

ii) Use the matrix in (i) above to find the basis for the image of T (2mks)

QUESTION TWO (20 MARKS)

a) i) Find a basis of the space $\mathbb{C}$ of complex numbers (2mks)

ii) Find the dimension of $\mathbb{C}$ (1mk)

b) Let $T : U \rightarrow V$ be a linear mapping where U *and* V are linear spaces

i) Define the image of T (1mk)

ii) Show that the kernel of T is a subspace of U (3mks)

c) Show that the matrices $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ form a basis for $M_{2\times 2}(\mathbb{R})$ (4mks)

d) Let $T: P_2 \rightarrow \mathbb{R}^{2\times 2}$ be defined by $T(a + bt + ct^2) = \begin{bmatrix} 1 & a \\ b & c \end{bmatrix}$. Determine whether T a linear transformation or not (3mks)

e) i) Define an invariant subspace (1mk)

ii) Let $T : U \rightarrow V$ be an isomorphism. If $f_1, f_2, \dots\dots f_n$ is a basis for U, show that $T(f_1), T(f_2), \dots\dots T(f_n)$ is a basis for V (5mks)

QUESTION THREE (20 MARKS)

a) State and prove Cayley Hamilton's theorem (8mks)

b) Let $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$

i) Find the eigenvalues of A and the associated eigenvectors (4mks)

UNIVERSITY OF KABIANGA

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

SECOND YEAR

SECOND SEMESTER (SUPPLEMENTARY) EXAMINATIONS

COURSE CODE: MAT 213

COURSE TITLE: **LINEAR ALGEBRA II**

Instructions to candidates:

Answers question ONE and any other TWO questions.

Time: 3 hours

QUESTION ONE - 30 MARKS (COMPULSORY)

a) Define the following terms in relation to linear spaces
   i) Linearly independent (1mk)
   ii) Basis (1mk)
   iii) Kernel (1mk)

b) i) Let $A = \begin{bmatrix} a & -b & c \\ -b & d & -e \\ c & -e & f \end{bmatrix}$. Show that A is a symmetric matrix (1mk)

   ii) Given that $\beta = \{(1,2,1),\ (2,9,0),\ (3,3,4)\}$ is a basis for $\mathbb{R}^3$, find the coordinate matrix of $T = (5,-9,9)$ with respect to β (4mks)

c) Let $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$.
   i) Find the eigenvalues of A (2mks)
   ii) For each eigenvalue, find the corresponding eigenvector (2mks)

d) i) Let $F: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear operator such that $F(x, y.z) = (2y + z,\ x - 4y,\ 3x)$. Find the matrix of F with respect to the basis $(w_1, w_2, w_3) = \{(1,1,1),\ (1,1,0),\ (1,0,0)\}$ (4mks)