

UNIVERSITY OF KABIANGA

MAT 314: ORDINARY DIFFERENTIAL EQUATIONS

BACHELOR OF EDUCATION & BACHELOR OF SCIENCE

SEMESTER I-2017-2018 ACADEMIC YEAR

**INSTRUCTIONS:** Answer question *ONE* and any other *two* questions

**QUESTION ONE (30 MARKS)-COMPULSORY**

- a) Explain the three properties used to classify differential equations (6 marks)
- b) Obtain the general solution of
- $ydx - xdy = xydx$  (6 marks)
  - $dy = (1 + y^2)e^x dx$  (3 marks)
- c) Find the particular solution of the differential equation below, that satisfy the given initial condition

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 1 = 0 \text{ given } y'(0) = \frac{1}{\sqrt{3}}, y\left(\frac{\pi}{6}\right) = 0$$

(5 marks)

- d) Define the following terms;
- Differential equation (2 marks)
  - Partial differential equation (2 marks)
  - Linear ordinary differential equation (2 marks)

- e) Classify the following equations
- $\dot{z} = xy + 3xy^2 - 4$  (2 marks)
  - $(2xy'')^2 + 4x^2y'' + 7xy' - 17xy = x^3$  (2 marks)

**QUESTION TWO(20 MARKS)**

- a. The rate of decay of Radium is proportional to the amount remaining. Prove that the amount remaining at any time  $t$  is given by  $A = A_0e^{-kt}$ , where  $A_0$  is the initial amount at time  $t = 0$  (6 marks)
- b. Solve  $\frac{dR}{dS} = \frac{1+R^2}{1-S}$ , given  $R(0) = 1$  (4 marks)
- c. Show that the equation  $(x^2 + y^2)dx + 2xydy = 0$  is homogenous and solve it. (10 marks)

**QUESTION THREE(20 MARKS)**

- a.
- i) State the necessary and sufficient condition that and equation of the form  $M(x, y) + N(x, y) = 0$  to be exact. (3 marks)
  - ii) Define the term integrating factor. (2 marks)
- b. Show that  $(3xy^4 + x)dx + (6x^2y^3 - 2y^2 + 7)dy = 0$  is exact and find the solution. (10 marks)
- c. Solve  $\frac{dy}{dx} = -\frac{3y^2+4x}{2xy}$  (5 marks)

**QUESTION FOUR(20 MARKS)**

- a. The room temperature in an office is  $70^\circ\text{F}$ . Experience has shown that the temperature of a cup of coffee brought to the office will drop from  $120^\circ\text{F}$  to  $100^\circ\text{F}$  in 10 minutes. What would be the temperature of the cup of coffee if it takes 20 minutes before it drops to  $100^\circ\text{F}$  (10 marks)
- b. Solve the equation  $y'' - 3y' + 2y = \frac{1}{1+e^{-x}}$  (10 marks)

**QUESTION FIVE(20 MARKS)**

- a. Use exponential shift to solve the problem  $(D^2 - 2D + 5)y = 16x^3e^{3x}$  (10 marks)
- b. Find the general solution of the non-homogenous system of equation
- $$\begin{aligned} \dot{X}_1 &= X_2 \\ \dot{X}_2 &= -2X_1 + 3X_2 \end{aligned}$$
- (5 marks)
- c. Show that  $(D - m)^n(x^k e^{mx}) = 0$  for  $k = 0, 1, \dots, n - 1$  (5 marks)