

UNIVERSITY OF KABIANGA

UNIVERSITY EXAMINATION

MAT 320:DYNAMICS EXAM FIRST SEM 2017/2018

INSTRUCTION:ANSWER ALL QUESTION IN SECTION A AND ANY TWO QUESTION IN SECTION B.

SECTION A:

Question one (30mks)

(a)The acceleration at any time t of a moving particle is given by $\frac{d^2r}{dt^2} = 12\cos 2ti - 8\sin 2tj + 16tk$.

(i)Find the velocity V and displacement r at any time t ,if at time $t=0$,it is known that $v=0$ and $r=0$
(6mks)

(b)A body is dropped (at rest)from a height of h metres.At what speed will it hit the ground
(6mks)

(c)Assuming two coordinates systems s and s' with coordinates x,y,z,t ,and x',y',z',t' respectively and Lorentz transformation given the relationship between the coordinates.

State three requirements that the transformation has to fulfill (3mks)

(d)Find the centre of mass of the system A,B,C,D if a mass of $1g$ is placed at $A(0,2)B(4,1)C(4,3)D(0,4)$.

Investigate the change in the centre of mass which takes place ^{if} at the mass at D is moved to $A(0,2)$ (8mks)

(e)State lagrange's equations giving all the symbols involved (4mks)

(f)A bead of mass m is constrained to move along a smooth rigid wire having the shape of the hyperbola $xy=c=\text{constant}$.Show that the kinetic energy may be expressed as $T=\frac{1}{2}m\dot{x}^2(1+\frac{c^2}{x^2})$
(3mks)

Question Two (20MKS)

(a) A particle moves along the curve $r = (t^3 - 4t)i + (t^2 + 4t)j + (8t^2 - 3t^3)k$

Where the parameter t denotes the time variable. Find the magnitude of its acceleration at $t=2$ (5mks)

(b) Show that the set of numbers of the form $a+b\sqrt{3}$ forms a group under multiplication (5mks)

(c) A uniform lamina is formed with boundaries given by the curve $y^2 = 4x (y \geq 0)$, the x -axis and the line $x=4$.

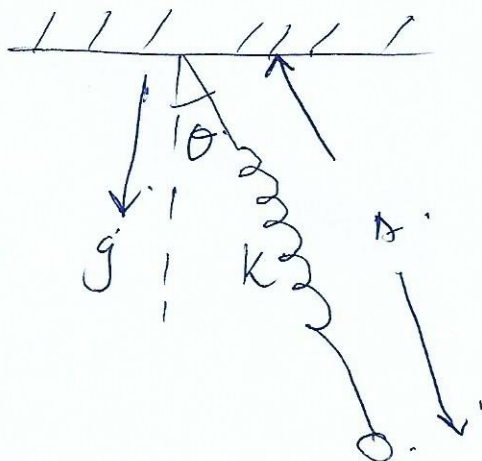
Find the position of the centre of mass. (10mks)

Question Three (20mks)

(a) Prove that in any principal ideal domains every ideal is a unique product of prime ideals (5mks)

(b) Calculate the coordinates of the centre of mass of particles of mass 4kg, 3kg and 2kg at points with coordinates (2,4), (0,3) and (5,1) relative to axes ox and oy . (5mks)

(c) For a pendulum bob suspended from a spring, find the equations of motion by a direct application of Hamilton principle. (10mks)

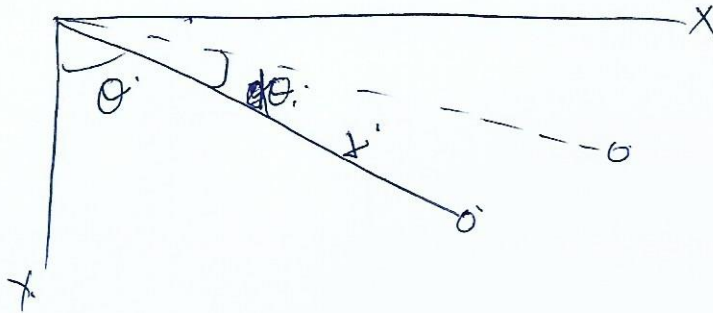


Question Four (20mks)

(a) Derive the equations relating the (x, y, z) and (r, θ, z) of a cylindrical coordinate system (4mks)

(b) Find the centroid of the region in the first quadrant that is bounded above by the line $y=x$ and below by the parabola $y=x^2$. (6mks)

(c) The figure below shows a pendulum bob attached to a rubber band.



Using r and θ as coordinates, obtain the generalized forces F_r and F_θ (10MKS)

Question Five (20mks)

(a) A particle moves along the curve $x=2t^2, y = t^2 - 4t, z = 3t - 5$, where t is the time. Find the velocity and acceleration at time $t=1$ (6mks)

(b) Derive the equations relating the (x,y,z) and (r,θ, ϕ) of a spherical coordinates system (4mks)

(c) A bead of mass m is free to slide along a smooth rigid parabolic wire the shape of which is given by $y=bx^2$, since the motion is confined to a line, the bead has only one degree of freedom.

These are two equations of constraint $y=bx^2$ and $z=c$. Obtain the completed equations of motion (10mks)