UNIVERSITY EXAMINATIONS 2017/2018
COURSE CODE: MAT 318

COURSE TITLE: FLUID MECHANICS I

INSTRUCTIONS

Answer **QUESTION ONE** (COMPULSORY) and **ANY OTHER TWO** questions.

Symbols used have their usual meaning in fluid mechanics.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Explain the following properties of fluids
   i) Compressibility
   ii) Specific heat
   iii) Enthalpy
   (6 Marks)

b) State the continuum hypothesis. (2 Marks)

c) Prove that for an ideal gas $C_p - R = C_v$ where C_p is specific heat at constant pressure, C_v is specific heat at constant volume and R is ideal gas constant. (9 Marks)

d) Show that the stream function ψ is a constant along a stream line. (4 Marks)

e) The three components of a fluid velocity in a flow field are given by:

$$u = x^2 + y^2 + z^2$$

$$v = xy + yz + z^2$$

$$w = -3xz - \frac{z^2}{2} + 4$$

   i) Show that this flow obeys the mass conservation equation. (4 Marks)
   ii) Determine whether the flow is irrotational or rotational. (5 Marks)

QUESTION TWO (20 MARKS)

a) The velocity in a certain flow field is given by the equation $v = 3x^2\vec{i} + 2x^2z\vec{j} + 5yz\vec{k}$ Determine the expressions for the three rectangular components of acceleration. (6 Marks)

b) The velocity component of a two dimensional flow are $u = \frac{x}{1+t}$, $v = \frac{-y}{1+t}$, determine the pathlines. (7 Marks)

c) Given the stream function for a flow as $\psi = 4(x^2 - y^2)$. Show that the flow is irrotational. (7 Marks)

QUESTION THREE (20 MARKS)

a) Derive the mass conservation equation for an ideal fluid by considering a fluid element. (9 Marks)

b) The stream function for a given two dimensional flow field is:

$$\phi = 5x^2 y - \frac{5}{3}y^3$$

Determine the corresponding ~~velocity potential.~~ ψ (6 Marks)

c) The velocity component of a two dimensional flow are $u = \frac{x}{1+t}$, $v = \frac{-y}{1+t}$ show that the streamlines are a constant. (5 Marks)

QUESTION FOUR (20 MARKS)

a) State Bernoulli's equation for incompressible fluid flow. (2 Marks)

b) Starting with Euler's Equation of motion derive Bernoulli's equation. (11 Marks)

c) A certain incompressible two-dimensional flow field has the velocity component in the y direction given by the equation:

$v = xy(3 + x)$.

Determine the velocity component in the x-direction so that the mass conservation equation is satisfied. (7 Marks)

QUESTION FIVE (20 MARKS)

a) Show that the streamlines for a flow whose velocity components are $u = c(x^2 - y^2)$ and $v = -2cxy$ where c is a constant are given by the equation $x^2 y - \frac{y^3}{3} = k$ where k is a constant. (8 Marks)

b) A flow field has the stream function $\psi = y(3x^2 - y^2)$

i) Determine velocity components. (4 Marks)

ii) Show that the flow is irrotational. (4 Marks)

iii) Show that this flow satisfies the equation of continuity. (4 Marks)