UNIVERSITY OF KABIANGA

UNIVERSITY EXAMINATIONS

MAT 214: VECTOR ANALYSIS EXAM 1ST SEM 2016/2017

INSTRUCTIONS: Answer ALL Questions in section A and **ANY TWO** in section B.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION (30MKS)

Question One (30mks)

a) If **A** = $5t^2$i + tj-t^3k and **B** =sin t i-costj find

(i) $\frac{d}{dt}$ (A.B) (3mks)

(ii) $\frac{d}{dt}$ (A X B) (3mks)

(iii) $\frac{d}{dt}$ (A .A) (3mks)

b) Find the area of the triangle having vertices at p(1,3,2) Q(2,-1,1) R(-1,2,3) (5mks)

c) If Ø (x, y, z) = $3x^2y - y^3z^2$, find ∇Ø at the point (1,-2,-1) (3mks)

d) If $\vec{A} = x^2 y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$, find the curl $\vec{A}$ (3mks)

e) Find the total work done in moving a particle in a force field given by

i−5zj

$\vec{F} = 3xy\hat{i} + 10X\,\hat{k}$, along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from t = 1 to t=2 (5mks)

f) Let $\vec{F}$= ~~$2xz\hat{i} + 10x\hat{k}$~~. Evaluate $2xzi - xj + y^2k$

$\iiint_V \nabla \cdot \vec{F}\, dv$, where V is the region bounded by the surfaces $x = 0, x = 2, y = 0, y = 6, z = x^2, z = 4$ (5mks)

SECTION B: ANSWER ANY TWO QUESTIONS IN THIS SECTION

Question Two (20mks)

a) Prove that a cylindrical coordinate system is orthogonal (9mks)

b) A particle moves along a curve such that its acceleration given parametrically is X=e^{-3t}, y=2 sin 3t, z=2 cos3t. Given that velocity and displacement were initially zero, find the full expression of the velocity and displacement at time t. (7mks)

c) Determine the value of a,b and c for which the vector

$\vec{r} = (x + 2y + az)\hat{\imath} + (bx + 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational. (4mks)

3-12 12 3 9

a) A particle moves so that its position vector is given by $\vec{r} = \cos\omega t\,\hat{\imath} + \sin\omega t\hat{\jmath}$ where ω is a constant. Show that

(i) The velocity $\vec{v}$ of the particle is perpendicular to $\vec{r}$ (5mks)

(ii) The acceleration $\vec{a}$ is directed toward the origin and has magnitude proportional to the distance from the origin (7mks)

(iii) $\vec{r}x\,\vec{v}$ = a constant vector (4mks)

b) If $A = 2\tilde{\imath} - 5\tilde{\jmath} + 3\tilde{k}, B = 3\tilde{\imath} - 4\tilde{\jmath} + 4\tilde{k}$ and $C = \tilde{\imath} + 3\tilde{\jmath} + 2\tilde{k}$. Find the unit vector parallel to $4A - 2B + C$but opposite in sense (4mks)

Question Four (20mks)

a) If $\varphi = 2xyz^2$, F = $xy\tilde{\imath} - z\tilde{\jmath} + x^2\tilde{k}$and C is the curve $x = t^2, y = 2t, z = t^3$ from t=0 to t=1

Evaluate the integrals

(i) $\int_c \varphi dr$ (4mks)

(ii) $\int_c Fdr$ (4mks)

b) If A = $(3x^2+6y)\tilde{\imath} - 14yz\tilde{\jmath} + 20xz^2\tilde{k}$, evaluate $\int c\ a.dr$ from (0,0,0) to (0,0,1),(0,1,1),(1,1,1) ~~(1,1,1)~~ along the following path C

(i) $x = t, y = t^2, z = t^3$ (4mks)

(ii) The straight line joining (0,0,0) then to (1,1,1) (4mks)

(iii) The strait line joining (0,0,0) and (1,1,1) (4mks)

Question Five (20mks)

a) If A $=\tilde{\imath}t - \tilde{\jmath}t^2 + \tilde{k}t\cancel{2}$ 3 and B $=\tilde{\imath}\sin t + \tilde{\jmath}\cos t$. Find the value of $\frac{d}{dt}(A.B)$ (2mks)

b) The acceleration of s particle at any time t≥ 0 is given by a = $\frac{d^2r}{dt^2}$ = $12\ cos2t\tilde{\imath} - 8\sin 2t\tilde{\jmath} + 16t\tilde{k}$if the velocity V and displacement r are zero at t =0, find V and r at any time (6mks)

c) Represent the vector A = $z\tilde{\imath} - 2x\tilde{\jmath} + y\tilde{k}$in cylindrical coordinates. Thus determine

$A_p,\ A_\varphi$ andA_z (5mks)

d) Verify Green's Theorem in the plane for $\oint_c (xy + y^2)\,\mathrm{dx} + \mathrm{x^2 dy}$ where C is the closed curve of the region bounced by y=x and y=x^2 (7mks)

2 6 8
3 3 4
4 1 φ

24 3
72

$\frac{3}{1} - \frac{14}{6} + \frac{20}{8} = \frac{72-56+60}{24}$

14
56

67 12
56
16

16+60

19
~~38~~
~~76~~
~~24~~
~~126~~