Pwani University College

A Constituent College of Kenyatta University

UNIVERSITY EXAMINATIONS 2010/2011 ACADEMIC YEAR

INSTITUTIONAL BASED PROGRAMME

3RD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

COURSE CODE/ TITLE: SMA 336: ORDINARY DIFFERENTIAL EQUATION II

END OF SESSION II DURATION: 3HRS

DAY/TIME: TUESDAY 4.00PM – 7.00PM DATE: 02.08.2011-EL

Instructions: Answer question ONE in Section A and any other two questions in Section B.

Section A

QUESTION ONE – (40 MARKS)

(a) Explain whether or not the differential equation $y''+y^3=4x$ is linear or not. **(2 marks)**

(b) Define linear independence and dependence of $f_1, f_2, f_3........f_n$ on the interval $[a,b]$ **(5 marks)**

(c) Prove that the functions $e^x \cos x$ and $e^x \sin x$ are linearly independent. **(5 marks)**

(d) Use the definition of Laplace Transform to work out the Laplace Transform of t^n **(4 marks)**

(e) By the method of elimination , introduce a differential operator D to solve the system

$$y'_1 - y_2 = x^2$$
$$y'_2 + 4y_1 = x$$

(6 marks)

(f) The function $y_1 = x$ is one of the solutions to $x^2 y'' - 4xy' + 4y = 0$ .Use the method of reduction of order to find the other solution. **(7 marks)**

(g) Prove that the transformation $x = e^t$ reduces second order Cauchy-Euler differential Equation $c_2 x^2 y'' + c_1 xy' + c_3 y = R(x)$ into linear differential equation of second order with constant coefficients. Hence solve $x^2 y'' - 2xy' + 2y = x^3$ **(11 marks)**

Section B

QUESTION TWO (15 MARKS)

(a) Use the Translation property of Laplace and the table below to answer the questions that follow.

Given the Laplace transforms

$$L(t^n) = \frac{n!}{s^{n+1}}, \qquad \mathrm{L}(\cosh at) = \frac{s}{s^2 - a^2},$$

$$\mathrm{L}(\sinh at) = \frac{a}{s^2 - a^2} \qquad L(\sin at) = \frac{a}{s^2 + a^2},$$

$$\mathrm{L}(\cos at) = \frac{s}{s^2 + a^2}, \qquad \mathrm{L}(e^{kt}) = \frac{1}{s-k}$$

all for $s > k$

find (i) $L\left\{e^{3t}\left(\cosh 2t - 3\sinh 5t\right)\right\}$ **(3 marks)**

(ii) $L^{-1}\left\{\dfrac{s+1}{s^2 - 6s + 13}\right\}$ **(5 marks)**

(b) By use of Laplace transforms, solve the initial value problem

$y''+2y'+5y=10$ given $y(0)=1,\quad y'(0)=0$ **(7 marks)**

QUESTION THREE (15 MARKS)

(a) Investigate whether or not the system below has solutions or not. If so predict the number of arbitrary constants expected in the solution.

$$y_1'+y_2'=1$$
$$y_1'+y_1+y_2'-y_2=0$$

(5 marks)

(b)

Solve the system of equations

$$y_1'-2y_1+2y_2'=2-4e^{2x}$$
$$2y_1'-3y_1+3y_2'-y_2=0$$

(10 marks)

QUESTION FOUR (15 MARKS)

(a) Verify that the vector function $y=\begin{pmatrix}4\\1\\1\end{pmatrix}e^{6t}$ is a solution to the system

$$y_1'=5y_1+2y_2+2y_3$$
$$y_2'=2y_1+2y_2-4y_3$$
$$y_3'=2y_1-4y_2+2y_3$$

(5 marks)

(b) Given the non-homogenous system $y'=\begin{pmatrix}2&0\\3&2\end{pmatrix}y+\begin{pmatrix}2\\1\end{pmatrix}e^{2t}$, use the method of undetermined coefficients to find the general solution of the system **(10 marks)**