

PWANI UNIVERSITY COLLEGE
UNIVERSITY EXAMINATIONS 2011/2012
Y3BK1 EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION ARTS &
SCIENCE

SMA200: CALCULUS II

3HOURS

INSTRUCTIONS

- This paper consists of *TWO* sections; **section A** and **section B**
- Answer *ALL* questions in section A and *any two* questions in section B

SECTION A [40 Marks]

Answer all the questions in this section

Question one (Compulsory)

- a) The set $P = \{0, 0.2, 0.6, 1, 1.5, 2\}$ is a partition of $[0,2]$. Find the norm $\|P\|$ of partition P. (4 marks)
- b) Use the fundamental theorem of Calculus to find $\frac{dy}{dx}$ if $y = \int_x^5 3t \sin t dt$. (4 marks)
- c) Evaluate: $\int (x + 1)(3x - 2) dx$. (4 marks)
- d) Solve: $\int x^2 \sin x^3 dx$. (4 marks)
- e) Evaluate: $\int \frac{1}{1+e^x} dx$. (4 marks)
- f) If $g(x) = \sin^2 x$, find the definite integral of $g(x)$ over $[0, 2\pi]$. (4 marks)
- g) Evaluate: $\int \frac{\log_2 x}{x} dx$. (4 marks)
- h) Evaluate: $\int x^2 e^x dx$. (4 marks)
- i) Use Trapezoidal rule with $n = 4$ to estimate $\int_1^2 x^2 dx$. (4 marks)
- j) Find the arc length of the graph of $f(x) = \frac{1}{2} x^2$ from $x = 0$ to $x = 1$. (4 marks)

SECTION B [30 Marks]

Answer any two questions from this section

Question two (Optional)

- a) Determine whether the indefinite integral $\int_{-\infty}^{\infty} x e^{-x^2} dx$ converges or diverges (7 marks)
- b) Approximate the value of $\int_0^1 3x^2 dx$ using Simpson's rule with $n = 4$ (3 marks)
- c) Show that $\int_0^{16} \frac{dx}{x^{\frac{5}{4}}}$ is divergent (5 marks)

Question three (Optional)

- a) Find the area of the region between the parabola $y^2 = 4x$ and the line $4x - 3y = 4$.
(7 marks)
- b) Compute the volume of a solid, which looks like a megaphone, obtained by revolving the line $y = 1 + \frac{x}{3}, 0 \leq x \leq 12$ about the x - axis (3 marks)
- c) Set up and evaluate the definite integral that gives the area of the region bounded by the graph of $f(x) = x^3$ and the tangent line to the graph at $(1,1)$ (5 marks)

Question four (Optional)

- a) Find the area of the surface formed by revolving the graph of $f(x) = x^2$ on the interval $[0, \sqrt{2}]$ about the y - axis (7 marks)
- b) Evaluate the improper integral: $\int_{-3}^1 \frac{dx}{\sqrt{1-x}}$ (3 marks)
- c) Integrate to find F as a function of x and demonstrate the second fundamental theorem of calculus by differentiating the result given that $F(x) = \int_0^x (t + 2) dt$ (5 marks)

Question five (Optional)

- a) An electric cable is hung between *two* towers that are 200m apart. The cable takes the shape of catenary whose equation is $y = 75 \left(e^{\frac{x}{150}} + e^{-\frac{x}{150}} \right) = 150 \cosh \frac{x}{150}$. Find the arc length of the cable between the *two* towers (7 marks)
- b) Find the arc length of the portion of the curve $y = x^2$ with $0 \leq x \leq 1$ (3 marks)
- c) A *four-leaf* clover is in the shape of the curve $r = \sin 4\theta$. Find its area (5 marks)

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COURSE OUTLINE

Riemann sums, fundamental theorem of calculus, integrating polynomial functions, integrating by substitution, integrating by exponential rule, integrating by log rule, integrating trigonometric functions, integration by parts, divergence and convergence of improper integrals, trapezoidal rule, Simpson's rule, area under curve, arc length, surface area of revolution and volume of solids