UNIVERSITY OF EMBU

2016/2017 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE, BACHELOR OF EDUCATION SCIENCE AND BACHELOR OF EDUCATION ARTS

SMA 320: METHODS 1

DATE: APRIL 6, 2017 **TIME: 11:00 AM-1:00 PM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE (30 MARKS)

a) Let $\Gamma(x)$ denote the Gamma function. Prove that

$$\Gamma(x+1) = x\Gamma(x) \qquad \text{for} \qquad x > 0$$ (5 marks)

b) Determine a power series solution for the initial value problem

$$y' - 2y = 0, \ \ y(0) = 3$$ (5 marks)

c) If $J_n(x)$ is Bessel's function of first kind of order n, prove that

$$J_{-n}(x) = (-1)J_n(x), \qquad \text{for } n = 1,2,3,.......$$ (5 marks)

d) Given the relation $f(t) * g(t) = L^{-1}\left(L(f(t) \cdot L(g(t))\right)$

Where $f(t)$ and $g(t)$ are piecewise continuous functions for $t \geq 0$, and that,

$|f(t)|$ and $|g(t)|$ are bounded, calculate the inverse Laplace transform of:

$$F(s) = \frac{1}{s(s^2+1)}$$ (5 marks)

e) Find the Fourier sine series for $f(x) = x$ on $-L \leq x \leq L$ (5 marks)

f) If $B(x, y)$ denotes Beta function in two variables x and y, prove that

$$B(x, y+1) = \frac{y}{x+y} B(x, y)$$ (5 marks)

QUESTION TWO (20 MARKS)

a) Prove the given formula well known as Bonnet's recurrence formula for Legendre polynomial.

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$$ (8 marks)

b) Calculate the convolution of $t^2 * \cos t$ (6 marks)

c) If $J_n(x)$ is Bessel's function of order n, prove that

$$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$$ (6 marks)

QUESTION THREE (20 MARKS)

a) If $\Gamma(x)$ and $B(x, y)$ denotes Gamma and Beta functions respectively,

i. show that the two functions have the following relation

$$B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$ (10 marks)

ii. Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ (5 marks)

b) Solve the partial differential equation using the method of separation of variables

$$\frac{\partial^2 z}{\partial x \partial y} = x^2 y$$ (5 marks)

QUESTION FOUR (20 MARKS)

a) Find the Fourier cosine series for $f(x) = x^2$ on $-L \le x \le L$ (8 marks)

b) Solve the following initial value problem using Laplace transform.

$$y'' - 2y = -4, \quad y(0) = 0, y'(0) = 0$$ (6 marks)

c) Determine a power series solution (up to 3rd terms) for the differential equation

$$y'' + xy = 0$$ (6 marks)

QUESTION FIVE (20 MARKS)

a) Solve the following initial value problem using Laplace transform.

$$x'' - 6x' + 34x = -34, \quad x(0) = 2, \ x'(0) = 3$$ (10 marks)

b) Show that any two different Legendre polynomials are orthogonal in the interval $-1 < x < 1$

(10 marks)

--END--