SOUTH EASTERN KENYA UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (PHYSICS)

SPH 306: MATHEMATICAL PHYSICS II

DATE: 16TH DECEMBER, 2016 **TIME: 4.00-6.00 P.M**

INSTRUCTIONS

1. This paper consists **of FIVE QUESTIONS**
2. Answer **QUESTION ONE** and **ANY OTHER TWO QUESTIONS**
3. Question ONE carries 30 marks while the other questions carry 20 marks each
4. All the symbols have their usual meaning
5. Tables are attached containing various relations which may be useful where necessary

QUESTION ONE (Compulsory) (30 marks)

(a) Find the Fourier series for

$f(x) = 1, \qquad -2 < x < 2$ **(7 marks)**

(b) Briefly explain the terms symmetry operation and symmetry elements **(5 marks)**

(c) One of the equations arising from Helmholtz equation $\nabla^2\psi + k^2\psi = 0$ in cylindrical system is

$$\frac{d^2Z}{dz^2} - \lambda_1 Z = 0, \qquad (\lambda_1 = \text{constant})$$ **(7 marks)**

Given that $\psi(r,\theta,z)$ vanishes at $z = 0$ and $z = L$. Show that

$$\lambda_1 = -\frac{n^2\pi^2}{L^2}$$

(d) Compute the Laplace transform of

$$f(t) = \begin{cases} 0, & 0 < t < a \\ 1, & a < t < b \\ 0, & b < t \end{cases}$$

(3 marks)

(e) Find the Fourier cosine and sine integrals for

$$f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & 1 < x \end{cases}$$

(8 marks)

QUESTION TWO (20 marks)

(a) By the use of Laplace transform solve the initial value problem

$$u'' + 4u' + 3u = 0, \qquad u(0) = 1, u'(0) = 0$$

(9 marks)

(b) Show that the Fourier sine transform of e^{-at} is

$$g_s(\omega) = \sqrt{\frac{2}{\pi}} \frac{\omega}{\omega^2 + a^2}$$

(11 marks)

QUESTION THREE (20 marks)

(a) Given a boundary value problem $\nabla^2 u = 0$, with the Laplacian operator in the spherical coordinate system as

$$\nabla^2 u = \frac{1}{\rho}\left\{\frac{\partial}{\partial \rho}\left(\rho^2 \frac{du}{d\rho}\right) + \frac{1}{\sin\phi}\frac{\partial}{\partial\phi}\left(\sin\phi \frac{\partial u}{\partial \phi}\right) + \frac{1}{\sin^2\phi}\frac{\partial^2 u}{\partial \theta^2}\right\}$$

and $\rho = \sqrt{x^2 + y^2 + z^2}$.

The variables are restricted by $0 \le \rho,\ 0 \le \theta < 2\pi,\ 0 \le \phi \le \pi$

Derive the Legendre differential equation

$$(1 - x^2)\frac{d^2 y}{dx^2} - 2x\frac{dy}{dx} + \mu^2 y = 0, \qquad -1 < x < 1$$

(11 marks)

where $x = \cos\phi,\ \ \Phi(\phi) = y(x),\ \ \mu^2$ is a constant

(b) In group theory,

(i) Define a group **(5 marks)**

(ii) Show whether or not the following set form a group

$[1, 0, -1]$ under arithmetic addition **(4 marks)**

QUESTION FOUR (20 marks)

Consider a quantity $J = \int_{x_1}^{x_2} f(y, y_x, x)dx$ that takes on an extreme value.

$f(y, y_x, x)$ is a known function, $y \equiv y(x),\ y_x \equiv \dfrac{dy}{dx}$.

(a) Derive the Euler equation $\dfrac{\partial f}{\partial y} - \dfrac{d}{dx}\dfrac{\partial f}{\partial y_x} = 0$ (12 marks)

(b) Given the integrand $f(y, y_x, x)$ to be of the form

$f(y, y_x, x) = f_1(x, y) + y_x f_2(x, y)$ (8 marks)

Show that the Euler equation leads to $\dfrac{\partial f_1}{\partial y} - \dfrac{\partial f_2}{\partial x} = 0$

QUESTION FIVE (20 marks)

Solve the initial value boundary value problem **(20 marks)**

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t} \qquad 0 < x < a,\ 0 < t$$

$$\frac{\partial u(0,t)}{\partial x} = 0,\quad u(a,t) = T_0, \qquad 0 < t$$

$$u(x,0) = T_1 \qquad 0 < x < a$$