# SOUTH EASTERN KENYA UNIVERSITY

## UNIVERSITY EXAMINATIONS 2016/2017

### FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (PHYSICS/ELECTRONICS/GEOLOGY/METEOROLOGY) AND BACHELOR OF EDUCATION (SCIENCE)

**SPH 204: MATHEMATICAL PHYSICS I**

**7TH DECEMBER, 2016** **TIME: 4.00-6.00 P.M**

**INSTRUCTIONS TO CANDIDATES**

- **This paper consists of FIVE questions.**
- **Answer question ONE and any other TWO questions.**
- **Question ONE carries 30 mark while the other TWO questions carry 20 marks each**

**QUESTION 1 [30 MARKS COMPULSORY]**

a. Calculate $\int_0^1 \tan^{-1} x dx$ [4 marks]

b. Evaluate $\int (4x^3 + 5x - 7) dx$ [2 marks]

c. Find $\int \sin^5 x \cos^2 x dx$ [4 marks]

d. Evaluate $\int \frac{2x^2 - x + 4}{x^3 + 4x^2} dx$ [4 marks]

e. integrate $\int (1 + \sin x)^5 \cos x dx$ [4 marks]

f. Approximate using arctanx the value of $\pi$ [3 marks]

g. Find the interval of convergence of $\sum_0^\infty \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots\dots\dots \frac{x^n}{n!} + \dots.$

[3 marks]

h. Show that $\frac{d^2y}{dx^2}=2\frac{dy}{dx}$, which has solution of the form $y=c_1+c_2e^{2x}$

[3 marks]

i. Compute the divergence of following vector field $\vec{F}(x,y,z)=\left[x^3y^3,3x^2,3zy^2\right]$

[3 marks]

**QUESTION 2 [ 20 MARKS]**

a. The earth is an oblate spheroid i.e. a sphere flattened at the poles whose radius is given by $r=a\left(1-\varepsilon\cos^2\Phi\right)$, where a is the equatorial radius and $a(1-\varepsilon)$ that the poles. Find the volume of the earth ( since ε is so small keep only the first order power in ε) [5 marks]

b. Show that $div(curl\vec{F})=0$ using the vector field $\vec{F}=yz^2\vec{i}+xy\vec{j}+yz\vec{k}$ [6 marks]

c. Given $f(x,y,z)=2x^2+3y^2+z^2$ and $\vec{a}=[1,0,-2]$ evaluate the direction of $\vec{a}$ at the point $[2,1,3]$ [4 marks]

d. Evaluate $\int\frac{x^2+2x-1}{2x^3+3x^2-2x}dx$ [4 marks]

**QUESTION 3 , 20 MARKS**

a. Calculate the total flux of the vector field with Cartesian components i.e. $\iint u..dA$ $u=(2x,y,-3x)$ out of the closed surface of the circular cylinder $x^2+y^2=a^2$ for $0\le z\le h$ [ 5 marks]

b. Determine whether $\vec{F}=x^2yi+xyzj-x^2y^2k$ is a conservative field [5 marls]

c. Find a unit normal vector of the cone of the revolution $z^2=4(x^2+y^2)$ at point p=(1,0,2) [5 marks]

d. Find $\int\tan^5\vartheta\sec^7\vartheta d\vartheta$ [5 marks]

**QUESTION 4, [20 MARKS]**

a. A curve between the points (0,0,0) and (1,1,1) is defined in the parametric form by $x=\lambda, y=\lambda^2, z=\lambda^2$ where the parameter λ run from 0 to 1. Calculate the vector field with Cartesian components $\int u..dl$ $u=(y-z,z-x,x-y)$ {5 marks]

b. Compute the curl of the following vector field $\vec{F}(x,y,z)=\left[e^x\cos y,e^x\sin y,0\right]$

[4 marks]

c. Find the particular solution of $y'''=0$ given that $y(0)=3, y'(1)=4, y''(2)=6$

[5 marks]

d. Evaluate $\int x\sin x dx$ [3 marks]

e. Show that the differential equation $\frac{dy}{dx}\ln x-\frac{y}{x}=0$ has the solution $y=c\ln x$

[3marks]

**QUESTION 5, [20 MARKS]**

a. Evaluate the integral $I=\iint_D x^2 y\,dx\,dy$ where D is a triangular region bound by the lines $x=0, y=0, x+y=1$ [5 marks]

b. Show that $y=c_1\sin 2x+3\cos 2x$ is a general solution for the differential equation of the form $\frac{d^2y}{dx^2}+4y=0$ [4 marks]

c. Find the general solution for the differential equation $dy+7x\,dx=0$ and a particular solution at $y(0)=3$ to find K [4 marks]

d. Find $\int \frac{1}{x^2\sqrt{x^2+4}}dx$ [4 marks]

e. Approximate $\frac{1}{1-0.02}$ to six decimal places [3 marks]

Some integrals you may need

$$\int x^n\,dx=\frac{x^{n+1}}{n+1}+C \quad (n\neq -1)$$

$$\int \frac{1}{x}\,dx=\ln|x|+C$$

$$\int e^x\,dx=e^x+C$$

$$\int a^x\,dx=\frac{a^x}{\ln a}+C$$

$$\int \sin x\,dx=-\cos x+C$$

$$\int \cos x\,dx=\sin x+C$$

$$\int \sec^2 x\,dx=\tan x+C$$

$$\int \csc^2 x\,dx=-\cot x+C$$

$$\int \sec x\tan x\,dx=\sec x+C$$

$$\int \csc x\cot x\,dx=-\csc x+C$$

$$\int \sinh x\,dx=\cosh x+C$$

$$\int \cosh x\,dx=\sinh x+C$$

$$\int \tan x\,dx=\ln|\sec x|+C$$

$$\int \cot x\,dx=\ln|\sin x|+C$$

$$\int \frac{1}{x^2+a^2}\,dx=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx=\sin^{-1}\left(\frac{x}{a}\right)+C$$

(1) $$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots, \qquad |x| < 1$$

(2) $$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \cdots, \qquad |x| < 1$$

(3) $$\frac{1}{1-2x} = 1 + 2x + 2^2x^2 + 2^3x^3 + 2^4x^4 + \cdots, \qquad |x| < \tfrac{1}{2}$$

(4) $$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \cdots, \qquad |x| < 1$$

(5) $$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 - \cdots, \qquad |x| < 1$$

(6) $$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots, \qquad |x| < 1$$

(7) $$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots, \qquad |x| < 1$$

(8) $$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

(9) $$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots$$

(10) $$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} + \cdots$$

(11) $$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \frac{x^9}{9!} + \cdots$$

(12) $$e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \cdots$$

**(13)** $$\int_0^x e^{-t^2}\,dt = x - \frac{x^3}{3} + \frac{x^5}{5 \cdot 2!} - \frac{x^7}{7 \cdot 3!} + \frac{x^9}{9 \cdot 4!} - \cdots$$

**(14)** $$\ln\left(\frac{1+x}{1-x}\right) = 2x + \frac{2x^3}{3} + \frac{2x^5}{5} + \frac{2x^7}{7} + \frac{2x^9}{9} + \cdots, \qquad |x| < 1$$

**(15)** $$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$$

**(16)** $$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

**(17)** $$(1+x)^p = 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \cdots, \qquad |x| < 1,$$

where $p$ is constant.