2601/203
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2603/203
ENGINEERING MATHEMATICS II
June/July 2016
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (POWER OPTION, TELECOMMUNICATION OPTION AND INSTRUMENTATION OPTION)

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have mathematical table/Non-programmable scientific calculator for this examination.
*Answer **FIVE** of the following **EIGHT** questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as shown.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) (i) Show that $\lim_{t \to 0} \frac{e^{-t} - e^{-2t}}{t} = 1$.

(ii) Hence find the Laplace transform of $f(t) = \frac{e^{-t} - e^{-2t}}{t}$. (7 marks)

(b) Use the convolution theorem to find the inverse Laplace transform of

$$F(s) = \frac{s}{(s^2 + 1)^2}$$

(6 marks)

(c) The circuit of figure 1 is dead prior to switch closure at t=0. Use Laplace transforms to find an expression for the current $i(t)$ for $t \geq 0$.

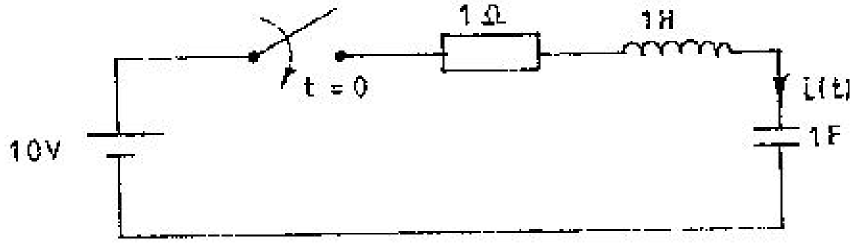

Fig. 1

(7 marks)

2. (a) Given the matrices $A = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -1 & 2 \\ 1 & 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, determine $C = A^2 - 4B$. (6 marks)

(b) Solve the equation $\begin{vmatrix} x & 3 & 2 \\ 1 & 1 & x \\ x & 1 & 2 \end{vmatrix} = 0$. (3 marks)

(c) Three emfs in a three-loop d.c. circuit satisfy the equations

$$3E_1 + 2E_2 - E_3 = 12$$
$$2E_1 + E_2 - 2E_3 = -12$$
$$E_1 - 2E_2 + 3E_3 = 10$$

Use the inverse matrix method to determine the values of the emfs. (11 marks)

3. (a) Given that $u = \frac{xy}{x + y}$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + 2\frac{\partial^2 u}{\partial x \partial y} = 0.$$

(7 marks)

(b) The power consumed in an electrical resistor is given by $P = \frac{E^2}{R}$ watts, where E is the voltage drop across the resistor and R is the resistance in ohms. Use partial differentiation to determine the change in power when E increases by 6% and R reduces by 0.1%, if the original values of E and R are 100 volts and 10 ohms respectively. (7 marks)

(c) Locate the stationary points of the function $z = x^2 + y^2 + 3xy - 3x - 2y + 8$, and determine their nature. (6 marks)

4. (a) Solve the differential equations

(i) $x^2 \frac{dy}{dx} = xy + x^2 + y^2$

(ii) $\frac{dy}{dx} + y \cot x = \cos x$ (12 marks)

(b) The response of a linear system is characterized by the differential equation

$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = e^{-t}$. Given that when $t = 0, x = 0$ and $\frac{dx}{dt} = 1$,

Use the D - operator method to solve the differential equation. (8 marks)

5. (a) Find the first three non-zero terms in the Maclaurin series expansion of f(x) =sin 2x, and hence evaluate $\int_0^1 \frac{\sin 2x}{x}\, dx$, correct to three decimal places. (10 marks)

(b) (i) Use Taylor's theorem to expand $\cos\left(\frac{\pi}{3} + h\right)$ as far as the term in h^4.

(ii) Hence determine the value of cos 63°, correct to four places of decimals. (10 marks)

6. (a) Find the value of the constant P given that the three vectors $A = 2i + 6j + pk$, $B = 4i + 5j - 6k$ and $C = -2i + j + 8k$ are coplanar. (5 marks)

(b) Given the magnetic field vector

$B = xy^2 i + x^2 yj + y^2 zk$, determine at the point (1, 2, -1)

(i) curl B;

(ii) div B (8 marks)

(c) The electric scalar potential in a region of space is given by $\phi = x^2 + xy^2 + z^2$ Determine, at the point (-1,2,-1)

(i) grad ϕ

(ii) the directional derivative of ϕ in the direction of the vector

$A = 2i + 2j - k$ (7 marks)

7. (a) Show that the general solution of the differential equation $\frac{dy}{dx} = xy + y^2x$ may be expressed in the form $y = A(1 + y)e^{\frac{1}{2}x^2}$. (9 marks)

(b) A machine member moves in such away that its displacement from a fixed position is given by the differential equation $\frac{d^2x}{dt^2} + 4x = 4\cos 2t$. Use the method of undetermined coefficients to solve the equation, given that when $t = 0$, $x = 0$ and $\frac{dx}{dt} = 2$. (11 marks)

8. (a) The lifetime of 100 light bulbs is normally distributed with a mean of 1570 hours and a standard deviation of 80 hours. Test the hypothesis that the mean lifetime of bulbs produced by the company is 1600 hours. (4 marks)

(b) A continuous random variable X has a probability density function f(x) defined by

$$f(x) = \begin{cases} K(\frac{1}{2}x + C), & 0 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

Given that the mean is $\frac{10}{9}$, determine:

(i) the values of the constants c and k;

(ii) the standard deviation;

(iii) $P\left(\frac{1}{2} < x < 1\right)$. (16 marks)

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