MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2017/2018

SECOND YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN APPLIED STATISTICS, ACTUARIAL SCIENCE, MATHEMTICAL SCIENCE AND MATHEMATICS AND ECONOMICS WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 200: CALCULUS II

Date: 21ˢᵗ February, 2018   Time: 3.30 - 6.30pm

INSTRUCTIONS:

- Answer Question ONE and any other TWO.

Question 1: Compulsory (30 marks)

a) Find the general solution of $G'(x) = \frac{1}{x^2}, x > 0$ and find a particular solution that satisfies the initial condition $G(1) = 0$. (3mks)

b) A company purchases a new machine for which the rate of depreciation is

$$\frac{dv}{dt} = 10,000(t-6), 0 \leq t \leq 5$$

where v is the value of the machine in pounds after t years. Set up and evaluate the definate integral that yields the total loss of value of the machine over the first 3 years. (4mks)

c) Evaluate the following integrals using the integration technique indicated after each question:

i) $\int \frac{x \cos \sqrt{3x^2+6}}{\sqrt{3x^2+6}} dx$; substitution method. (4mks)

ii) $\int (\sin^2 3x \cos 3x) dx$; change of variables method. (4mks)

iii) $\int e^x \sin x dx$; integration by parts method. (7mks)

iv) $\int \frac{x^3 + x^2 - 2x + 1}{x^2 - 5x + 6} dx$; use partial fractions (8mks)

Question 2 (20 marks)

a) Using Shell Method, calculate the volume swept by the space between $y = -x^2 + 4x$ and $y = x^2 - 4x + 6$ about $y-$axis. (7mks)

b) Show that the surface area, A, of a sphere of radius r units is given by the formula $A = 4\Pi r^2$. (9mks)

c) Evaluate the following integral involving absolute values:

$$\int_0^2 |2x - 1| \, dx$$

(4mks)

Question 3 (20 marks)

a) State the Fundamental Theorem of Calculus and give its proof. (8mks)

b) Calculate the area bounded by the curve $y = 3 - x^2$ and the line $y = -1$

   i)by integrating with respect to x and

   ii)by integrating with respect to y. (8mks)

c) Find the mean value of $i(t) = 20 + 2\sin(\Pi t)$ for $t = 5$ to $t = 0.5$. (4mks)

Question 4 (20 marks)

a) Using the disk method show that the volume, V, of a sphere of radius r is given by the formula

$$V = \frac{4}{3}\Pi r^3$$

(6mks)

b) Use the trapezoidal rule to approximate

$$\int_0^{\Pi} \sin x\, dx$$

for $n = 4$ and $n = 8$ and by finding the exact area compare the results. (10mks)

c) Evaluate $\int \sin^5 x \cos^3 x\, dx$. (4mks)

Question 5 (20 marks)

a) A ball is thrown vertically upwards with an initial velocity of 60 feet per second. How high will the ball go. (Take $a(t) = -32 ft/sec^2$ as the acceleration due to gravity). (10mks)

b) Calculate the length traced by a particle $p(x, y)$ from time $t = 0$ to $t = \frac{\Pi}{2}$ when $x = \sin^2 t$ and $y = \cos^2 t$ (6mks)

c) Find $\int_1^3 x^2 dx$ using Simpson's Rule with $h = 0.5$ (4mks)

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