UNIVERSITY EXAMINATIONS

2 (c)

FIRST SEMESTER 2014/2015

MAT 110: BASIC CALCULUS

INSTRUCTIONS: Answer **QUESTION ONE** (COMPULSORY) and **ANY OTHER TWO** Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Evaluate the following limits

i) $\lim\limits_{x \to 2} 7x^2 - 3x + 11$ (2 Marks)

ii) $\lim\limits_{x \to 1.5} \dfrac{16x^2 - 36}{4x - 6}$ (3 Marks)

b) Obtain the derivative of the function $y = \sqrt{2x - 4}$ with respect to x from first principles. (4 Marks)

c) Find the equation of the tangent line to the curve $y = 4x^3 - 5x^2 + 2$ at the point $x = 0.5$ (5 Marks)

d) Find $\dfrac{d^2y}{dx^2}$ for the curve implicitly defined by the functions:

i) $5xy + x^2 + 2y^2 = 10$ (2 Marks)

ii) $x^3 + 7x^2y^2 - 4y^3 = 0$ at (1,1) (3 Marks)

e) Evaluate the following integrals

i) $\int_0^2 x^2(3x - \sqrt{x})dx$ (3 Marks)

ii) $\int_0^\pi 1 + 3Cosx + e^x dx$ (3 Marks)

f) If $\dfrac{dy}{dx} = 20x^3 - 12x^2 + 5$ for a particular curve, and it is known that y = 40 when x = 2. Determine y in terms of x (5 Marks)

$\frac{d}{dx}(20x^3 - 12x^2 + 5)$

$= 20 \cdot 3x^2 - 12 \cdot x^1 + 0$

$= 60x^2 - 12x$

QUESTION TWO (20 MARKS)

a) Find $\frac{dy}{dx}$ given that $y = \sqrt{\frac{3+9x^2}{5x-8}}$ (4 Marks)

b) Given the curve $y = 2x^2 - 3x + 1$ at $x = 3$ determine the equations of:

i) the tangent. (4 Marks)

ii) the normal. (3 Marks)

c) Find the stationary points of $y = 2x^3 - 2x^2 - 2x + 1$ and determine their nature (9 Marks)

QUESTION THREE (20 MARKS)

a) Find $\frac{dy}{dx}$ given that $y = t^4 \tan 4t$ and $x = 3Sin2t$ (5 Marks) parametric

b) Given the function: $f(x) = x - k/x$

i) For what value of k will $f(x)$ have a stationary point at $x = -2$? (3 Marks)

ii) Determine the nature of the stationary point in i). (2 Marks)

c) Evaluate the following integral:

$$\int_1^4 \sqrt{x}\left(x^2 - 5\right)dx$$ (5 Marks)

d) Find the area under the curve $y = x^2(x-2)$ from $x = 2$ to $x = 5$ (5 Marks)

QUESTION FOUR (20 MARKS)

a) Show that $\frac{d(Sinx)}{dx} = Cosx$ from first principles (5 Marks)

b) The acceleration of a particle t seconds after it starts moving is $(10t^2 - 3t)m/s^2$. The particle started with a velocity of 4 m/s. Find:

i) The velocity of the particle in terms of t. (3 Marks)

ii) The displacement of the particle from its starting point in terms of t. (3 Marks)

iii) It's velocity after 2 seconds. (2 Marks)

c) Find the area bound by the curves $y = 8 - x^2$ and $y = x^2$ (7 Marks)

QUESTION FIVE (20 MARKS)

a) Differentiate the following functions with respect to x:

i. $y = 3Cos(x^2 - 10)$ (3 Marks)

ii. $y = 5x^3 Tan 8x$ (3 Marks)

b) Evaluate the following integrals

i. $\int Sin(2x+1)dx$ (3 Marks)

ii. $\int 3e^{5x-8}dx$ (3 Marks)

c) Find the area of the region bounded by the parabola $y^2 = 4x$ and the line $y = 2x - 4$ (8 Marks)

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$\frac{dy}{dx} = 6x^2 - 4x - 2 = 0$

$6x^2 - 4x - 2 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{4 \pm \sqrt{16+48}}{2 \times 6}$

NOT INCLUSSIVE!!

NOT INCLUSSIVE

NOT

$x = 1$ or $-\frac{1}{3}$

at $x = 1$

$y = 2(1)^3 - 2(1)^2 - 2(1) + 1$

$= -1$

point $(1, -1)$ -- Point (A)

x	0	1	2
$\frac{dy}{dx}$	-2	0	14
sign	\	—	/

minimum

at

$x = -1/$