

MAT 110: BASIC CALCULUS SEMESTER I 2015/2016
DEC 2015 EXAMS

INSTRUCTION TO CANDIDATE

- This paper consists of **FIVE** questions.
- Question **ONE** is compulsory. Attempt question **ONE** and **ANY** other **TWO** questions.

QUESTION ONE (30 MARKS) COMPULSORY

- a) Define a function and name the different kinds of variables [4Mks]
- b) State the difference between a domain and a range [4Mks]
- c) If $f(x) = \frac{5}{x-2}$ and $g(x) = 2x + 1$.
- i) Compute $(f \circ g)(x)$ [4Mks]
 - ii) Use your answer in i) to solve $(f \circ g)(3)$ [4Mks]
 - iii) State the domain and the range of f, g and $f(g(x))$ [3Mks]
- d) State the three conditions for a function to be said to be continuous [3Mks]
- e) Show that $\frac{d}{dx}[af(x) + bg(x)] = af'(x) + bg'(x)$ [4Mks]
- f) Find $\frac{d}{dx}[(1 - x - 2x^2 + 6x^3)(4x^2 - 6x + 2)]$ [4Mks]

QUESTION TWO (20 MARKS)

- (a) The area of a rectangle cattle pen is to be 64m^2 . If x meters is the length of one side, show that the perimeter of the rectangle is $\left(2x + \frac{128}{x}\right)$ meters. What is the minimum perimeter of the pen? [5Mks]
- (b) Given $h(x) = \frac{2x^3 + 4}{x^2 - 4x + 1}$, find $h'(x)$. [5Mks]
- (c) Evaluate the following limits
- (i) $\lim_{x \rightarrow \infty} \frac{3x + 4}{2x^2 - 5}$ [5Mks]
 - (ii) $\lim_{t \rightarrow \infty} \frac{f(2+t) - f(2)}{t^2}$, where $f(x) = x^2$. [5Mks]

f'x g'x

log x = log 6

x² = 64

x² = 4³

QUESTION THREE (20 MARKS)

- (a) Find the equation of the line tangent to the curve, $x = t^2 - 2$, $y = t^2 - 2t + 1$, at the point where $t=2$. [5Mks]
- (b) Given $y = e^{2x+\ln x}$, find $\frac{dy}{dx}$. [2Mks]
- (c) Find an equation of the tangent line to the curve $x^3 + y^3 = 9$ at the point $(1, 2)$. [5Mks]
- (d) Find the nature of the turning points and hence sketch the curve $y = x^3 - 6x^2 + 9x$. [8Mks]

QUESTION FOUR (20 MARKS)

- (a) Evaluate $\int_{-1}^2 (2x^3 - 3x^2 + x - 1) dx$ [5Mks]
- (b) Given $f(x) = \frac{1}{\sqrt{x}}$ find the derivative of f using:
i). The first principle [4Mks]
ii). The power rule [3Mks]
iii). The reciprocal rule [3Mks]
- (c) Evaluate $\int \frac{\tan x}{\cos^2 x} dx$ [5Mks]

$$\frac{-f'(x)}{(f(x))^2}$$

QUESTION FIVE (20 MARKS)

- (a) The acceleration of a particle t seconds after it starts moving is $(3t - 2)m/s^2$. The particle started with a velocity of 5 m/s. Find:
i) The velocity of the particle in terms of t . [3Mks]
ii) The displacement of the particle from its starting point in terms of t . [3Mks]
iii) It's velocity after 3 seconds. [4Mks]
- (b) Find the slope of the tangent to the curve $x^2 + \frac{x}{y} + y^2 = 7$ at the point $(1, 2)$. [5Mks]
- (c) Discuss the continuity of the function $f(x) = \frac{1}{x-2}$ [5Mks]