**JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY**

**UNIVERSITY EXAMINATIONS 2015/2016**

**YEAR II SEMESTER I EXAMINATION FOR THE DEGREE OF BACHELOR OF MATHEMATICS AND COMPUTER SCIENCE**

**SMA 2231- PROBABILITY AND STATISTICS III**

**DATE: DECEMBER 2015 TIME: 2 HOURS**

**INSTRUCTIONS: ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS**

**QUESTION ONE – 30 MARKS**

1. Determine the value of K that makes the function f(x,y) = k|x-y| a joint probability mass function over the points x=-2,0,2 and y=-2,3 [3 marks]

(b)(i) Define the term marginal distribution of continuous random variables x and y [2 marks]

 (ii) Let X and Y have the joint p.d.f defined by f(x,y) = Ce-2x-3y, o<x<, 0<y<x. determine

 the value of c and the marginal distribution of x [7 marks]

(c )(i) What is meant by independence of two random variables x and y [1 mark]

 (ii) Let X and Y have the following p.m.f

|  |  |  |  |
| --- | --- | --- | --- |
| x/y | 1 | 2 | 4 |
| 2 | 0.10 | 0.20 | 0.20 |
| 3 | 0.04 | 0.05 | 0.08 |
| 5 | 0.006 | 0.15 | 0.12 |

 Are X and Y independent? [4 marks]

(d)(i) Define the term expected value of a random variable ~~Z~~ = g(x,y) [2 marks]

 (ii) Suppose that x and y are independent random variables. Let g(x) and h(y) be functions

 of x and y respectively. Prove that E[g(x).h(y)] = [Eh(y)] [4 marks]

(e )(i) A random variable x has the p.d.f.

  Find the p.d.f. of the random variable y=e-x [4 marks]

 (ii) Suppose that x and y are independently distributed random variables each having a poisson distribution with parameter. Show that x+y has a poisson distribution with parameter 2 [3 marks]

**QUESTION TWO – 20 MARKS**

1. Suppose that x and y have the following p.m.f.

|  |  |  |  |
| --- | --- | --- | --- |
| x\y | 0 | 1 | 2 |
| 0 | 3/28 | 9/28 | 3/28 |
| 1 | 3/14 | 3/14 | 0 |
| 2 | 1/28 | 0 | 0 |

Find the covariance of x and y [7 marks]

1. Let x and y be two random variables and let a and b be any two constants. Prove that Var(ax + by) = a2 var(x) + b2 var(y) + 2ab cov(x,y) [7 marks]

 Given the joint p.d.f. (f(x,y) = {6-x-y, 0<x<2, 2<y<4

 8

 0, otherwise

 Find P(1<y<3/x=2) [6 marks]

**QUESTION THREE – 20 MARKS**

1. Let the random variable x have the standard normal distribution. Find the distribution of the random variable y = x2 [6 marks]
2. Suppose that the life of a certain light is exponentially distributed with mean 100 hours. If 10 such light bulbs are installed simultaneously, what is the distribution of the life of the light bulb that fails first and what is its expected life? [6 marks]
3. If x1, x2 ….xn are independent random variables and let the m.g.f for each exist for

 n

all -h<t<h, H>0. Let y = $\sum\_{i=1}^{n}xi$. Prove that My(t) = Mxi(t). [3 marks

 c=*i* 

1. Let x1 and x2 be jointly continuous with p.d.f. f(x1, x2) = , x, >0, x2 >0. Let

y1= x1 + x2 and y2 = . Find the joint distribution of y1 and y2 [5 marks]

**QUESTION FOUR – 20 MARKS]**

(a)(i) Define the term order statistics [2 marks]

 (ii) Let y, < y2 < y2 <y3 <y4 <y5 denote the order statistics of a random sample of size n=5

 from a distribution having p.d.f. f(x) = {e-x 0<x<

 0, elsewhere

 Find the p.d.f. of the minimum y1 and y3 [4 marks]

(b) Suppose the joint p.m.f. of x and y is given by fxy(x,y) =  for x = 0,1,2,3 and y= 0,1,2

 Find

1. p(x<2, y = 1) [2 marks]
2. p(x>2, y< 1) [2 marks]
3. p(x>y) [2 marks]
4. p(x + y = 4) [2 marks]

(c) Let x and y have the joint p.d.f.

 Fxy (x,y) = cxy, 0<x<3, 0<y<3

 0, otherwisw

 Find

1. the value of c [3 marks]
2. p(x<2, y<3) [3 marks]