**JOMO KENYATTA UNIVERSITY**

**OF**

**AGRICULTURE AND TECHNOLOGY**

 **UNIVERSITY EXAMINATIONS 2015/2016**

**YEAR III SEMESTER II EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN INFORMATION TECHNOLOGY**

**SMA 2230: PROBABILITY AND STATISTICS II**

**DATE: APRIL 2016 TIME: 2 HOURS**

**INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS. Z – VALUE AND STUDENT’S T – DISTRIBUTION TABLES WILL BE PROVIDED**

**QUESTION ONE (30 MARKS)**

1. Define random variable. [2 marks]
2. A random variable X has the distribution function shown below;

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| F(X) | K | 3k | 5k | 7k | 9k | 11k | 13k | 15k | 17k |

 K is a positive constant, find;

1. The value of K [2 marks]
2. The probability distribution of X [2 marks]
3. P (3 ≤ x ≤ 6) [2 marks]
4. In a certain community the probability of female birth is 0.6. If 10 individuals are randomly selected find;
5. The probability that exactly 4 of them are females. [2 marks]
6. The expected number of females in the sample. [2 marks]
7. The probability density function of a random variable X is given by;

 $f\left(x\right)=\{\_{0}^{e^{-x}}$ for x ≥ 0 otherwise

Find;

1. E [x] [3 marks]
2. The moment generating function of X [3 marks]
3. A random variable X is normally distributed with a mean of μ and variance$μ^{2}$. Given that p(x≤8) = 0.95, determine p (6≤ x ≤ 9). [4 marks]
4. A machine produces bolts which are 10% defective. Find the probability that in a random sample of 400 bolts produced by the machine, at most 30 of the bolts will be defective. [3 marks]
5. According to the National office of vital statistics of Ministry of Health and Human services, the average number of accidental drowning per year in Kenya is 3 per 100,000 people. In a certain town the population is 400,000.

(i) Justify the use of a Poisson approximately for the distribution of the number of

 drownings per year in this town. [2 marks]

(ii) Find the probability that in this town there will be not more than 3 accidental

 drownings. [3 marks]

**QUESTION TWO (20 MARKS)**

1. A discrete random variable X has the probability mass function (pmf)

 P(x) = $\{\_{0}^{K (x+1)}$ for x = 0, 1, 2, 3, 4 elsewhere

(i) Find the value of the constant K. [2 marks]

(ii) Calculate the expectation and variance of the distribution. [4 marks]

(iii) Find probability that X exceeds the expected value. [2 marks]

1. For the binomial random variable X, the number of trials n = 10 and the probability of success for any trial S = 0.35.

(i) Determine mode of the distribution. [2 marks]

(ii) Determine p(x≤3) correct to 4 d.p. [2 marks]

(iii) Determine median of the distribution [2 marks]

1. Write down the binomial probability generator and hence show that :-

(i) μ = n s [3 marks]

(ii) $σ^{2}$ = nsf [3 marks]

**QUESTION THREE (20 MARKS)**

1. Let y = ax + b, where a and b are real constants. Show that ;-

(i) E[Y] = a E [x] + b [2 marks]

(ii) Var [Y] = $a^{2}$ Var [x] [2 marks]

1. The mean weight of 500 students at a certain college is 49 Kg and the standard deviation is 5 Kg. Assuming that the weights of the students normally distributed, find how many students weigh;

(i) Between 45 Kg and 55 Kg [4 marks]

(ii) More than 60 Kg [4 marks]

1. The table below shows the probability distribution of random variable X.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X | 0 | 1 | 2 | 3 | 4 |
| F (x) | 0.125 | 0.375 | 0.485 | 0.85 | 1.00 |

Find the;

(i) Distribution of function of x. [2 marks]

(ii) Expected value of x [3 marks]

(iii) Standard deviation of x [3 marks]

**QUESTION FOUR (20 MARKS)**

(a)(i) Define a normal distribution of a random variable. [2 marks]

 (ii) A random variable x is normal random variable with μ = 30 and $σ^{2}$ = 25, i.e N(30,25)

 Compute p (25 ≤ x ≤ 42) [5 marks]

(b) The moment generating function of a normal random variable is given by 5t x 2$t^{2}$

 Mx(t) = 

 Compute p (8 ≤ x ≤ 10) [5 marks]

(c)(i) Define a hypergeometric distribution of a random variable x. [2 marks]

 (ii) A shipment has 100 tape recorders, of which 10 of these units are defective. What is

 the probability that among 5 tape recorders chosen for inspection two will be

 defective? [3 marks]

(d) At a certain supermarket 40% of the customers pay by credit card. Find the probability

 that in a randomly selected sample of 10 customers, 6 or more pay credit card. [3 marks]

**QUESTION FIVE (20 MARKS)**

1. A sample size of 80 has $\overbar{x}$ = 50 gm with a standard deviation σ = 5 gm. Find a 99% confidence interval limits for the population mean. [3 marks]
2. (i) State without proof the central limit theorem. [2 marks]

(ii) A father wishing to keep his son busy offers to pay Shs.20 if the mean score of 100

 throws of a die exceeds 4. What is the probability that his father will pay? [6 marks]

(c)(i) Define the term student t – distribution. [2 marks]

 (ii) The widths of a sample of 10 specimens, chosen from a particular locality were measured and found to be 26, 19, 24, 14, 16, 11, 13, 20, 17, 20. Previous extensive measurements of the specimen of the sample species had shown them to be normally distributed with a mean of 21 cm. Using a 5% significance level, two – tail, test whether the specimen from the chosen locality have different widths from the main population. [7 marks]