MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

THIRD YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 319: NUMERICAL MATHEMATICS

Date: 2nd December, 2016 | Time: 12.00 - 3.00 pm

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.
- Start each question on a fresh page.
- Indicate question numbers clearly at the top of each page.
- Observe further instructions on the answer booklet.

Question One [30marks]

a) Use partial pivoting to evaluate the determinant of the ma

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a) Use partial pivoting to evaluate the determinant of the matrix. [7mks]

$$\begin{pmatrix} 1 & 1 & 0 & 3 \\ 2 & 1 & -1 & 1 \\ -1 & 2 & 3 & -1 \\ 3 & -1 & -1 & 2 \end{pmatrix}$$

b) Given the system

$$\begin{aligned} 4x - y + z &= 7 \\ 4x - 8y + z &= -21 \\ -2x + y + 5z &= 15 \end{aligned}$$

Solve using the following methods with initial values [1, 2, 2].

i) Jacobi iteration [4mks]

ii) Gauss-Seidel iteration [4mks]

giving the first three iterates to 4 decimal places.

c) Find the triangular factorization $A = LU$ (Doolittle method) where

$$A = \begin{pmatrix} 2 & 4 & -6 \\ 1 & 5 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

hence solve for $\mathbf{x}$ in $A\mathbf{x} = \mathbf{B}$ where $\mathbf{B} = [-4, 10, 5]^t$. [6mks]

d) Show that the Newton-Raphson formula for the N^{th} root of A is [4mks]

$$X_{n+1} = \frac{(N-1)X_n - \frac{A}{X_n^{N-1}}}{N}$$

e) Use Euler's method to solve IVP

$$\frac{dy}{dt} = -y + t + 1, \quad 0 < t < 0.5, \ y(0) = 1$$

take $h = 0.1$ [5mks]

Question Two [20 marks]

a) Use the secant method to find a positive root to 4 decimal places of

$$f(x) = x^3 - 4x^2 + x - 10$$

[10mks]

b) Given

$$A = \begin{pmatrix} a+3 & 2 \\ 2 & a \end{pmatrix}$$

Find the eigenvalues and the corresponding eigenvectors. [10mks]

Question Three [20 marks]

The iteration $\mathbf{X}^{(k+1)} = \mathbf{X}^{(k)} + A\mathbf{f}(\mathbf{X}^k)$ where $\mathbf{X}$ is a vector, $\mathbf{f}$ is a vector of functions and A is a constant matrix, converges to a root of $\mathbf{f}(\mathbf{X}) = \mathbf{0}$. Given

$$X^{(0)} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\mathbf{f}(\mathbf{X}) = \begin{pmatrix} x^2 + y^2 - 9 \\ x - y^2 + 1 \end{pmatrix}$$

$$A = -\frac{1}{10}\begin{pmatrix} 2 & 2 \\ 1 & -2 \end{pmatrix}$$

Use the above algorithm to find a solution of $\mathbf{f}(\mathbf{X}) = \mathbf{0}$ near $\mathbf{X}^{(0)}$ correct to 2 decimal places. [20mks]