MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FOURTH YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 404: COMPLEX ANALYSIS II

Date: 5th December, 2016 Time: 8.30 - 11.30 am

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.
- Show all the necessary workings
- Start each question on a new page.
- Observe further instructions on the answer booklet.

QUESTION ONE (COMPULSORY) (30mks)

(a) Show that

(i) $|z_1 + z_2| \leq |z_1| + |z_2|$ [3mks]

(ii) Let $||z_1| - |z_2|| \leq |z_1 - z_2|$ [2mks]

for all $z_1, z_2 \in \mathbb{C}$

(b) Find a set of all the six distinct roots of the equation $z^6 = -i$, where $i = \sqrt{-1}$ [3mks]

(c) Show that the function $f(z) = z^3 + e^z$ satisfies the Cauchy-Riemann equations and hence obtain the derivative $f'(z)$ from the above equations involving partial derivatives. [3mks]

(d) Evaluate the integral

$$\int_C \frac{z+2}{(z^2-25)(z+2i)} dz$$

where C is the circle $|z| = 3$ described in the positive sense. [4mks]

(e) State Cauchy's Integral theorem for a multiply connected domain (say a triply connected domain). [2mks]

(f) Explain the following notions:

(i) $f(z)$ is analytic at a point z_0 in its domain. [2mks]

(ii) Singular point of a function [1mk]

(iii) Residue of a function $f(z)$ at an isolated singular point [3mks]

(iv) $f(z)$ has a removable singularity at z_0 [3mks]

(v) $z = a$ is a pole of order m of the function $f(z)$ [2mks]

g) State the Cauchy's Residue Theorem [2mks]

QUESTION TWO (20mks)

(a) Let $f(z)$ be analytic in a simply connected domain D bounded by a rectifiable Jordan curve C and be continuous on C. Show that

$$f(z) = \frac{1}{2\pi i} \int_C \frac{f(w)}{w-z} dw$$

for all $z \in D$ [10mks]

(b) In part (a), show that

$$f'(z) = \frac{1}{2\pi i} \int_C \frac{f(w)}{(w-z)^2} dw$$

at any point $z \in D$. State without proof, the formula for $f^n(z)$ (the n^{th} derivative of $f(z)$, where n is any positive integer > 1). [10mks]

QUESTION THREE (20mks)

Evaluate(using calculus of residues)

(a) $\int_0^{2\pi} \frac{1}{(a+b\cos\theta)^2} d\theta$ where $0 < b < a$

(b) $\int_0^{\infty} \frac{x^2}{(x^2+1)^2} dx$ [20mks]

QUESTION FOUR (20mks)

(a) If $f(z)$ is analytic in the doubly connected region defined by $\rho <\mid z-a \mid< R$. Show that $f(z)$ can be expressed in a series

$$f(z) = \Sigma_{n=-\infty}^{\infty} a_n (z-a)^n$$

where the $a_n' s$ are constants. [10mks]

(b) Find the Taylor or Laurent series which represent the function

$$f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$$

when

(i) $\mid z \mid< 2$

(ii) $2 <\mid z \mid< 3$

(iii) $\mid z \mid> 3$ [10mks]

QUESTION FIVE (20mks)

(a) State and prove Liouville's theorem [10mks]

(b) Show that every polynomial of degree $n \geq 1$ has at least one zero. [10mks]