MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FOURTH YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 405: PARTIAL DIFFERENTIAL EQUATIONS I

Date: 29th November, 2016 Time: 8.30 - 11.30 am

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.
- Show all the necessary workings
- Start each question on a new page.
- Observe further instructions on the answer booklet.

MASENO UNIVERSITY

MMA 405 – PARTIAL DIFFERENTIAL EQUATIONS (P.D.E.) I
4^{TH} YEAR, 1^{ST} SEM (2016/2017)

INSTRUCTION: ANSWER QUESTION **ONE** AND ANY OTHER TWO QUESTIONS

QUESTION ONE (COMPULSORY)

a) Define the following terms:
   i. Partial differential equation(P.D.E)
   ii. Linear P.D.E
   iii. A general solution of P.D.E

(3marks)

b) Form a P.D.E. from the following relations
   i. $z = ax^3 + by^3$
   ii. $z = ax^3 + bx^2y + cxy^2 + d\frac{y^4}{x}$

(6 marks)

c) Solve the following P.D.E.

$$(x^2 - y^2 - z^2)p + 2xyq = 2xz.$$

(4 marks)

d) Use Lagrange's method to solve

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + t\frac{\partial z}{\partial t} = mz + \frac{xy}{t}$$

(5 marks)

e) Show that $x + 1 = 3 - 0.5y = z$ lies in the plane $x + \frac{1}{3}y = 1 + \frac{1}{3}z$ (3 Marks)

f) Solve $px + qy + k(1 + p^2 + q^2)^{1/2} = z$

(4 marks)

g) Determine the solution of $16p^2z^2 + 9q^2z^2 + 4z^2 - 4 = 0$ which takes the form of $f(z, p, q) = 0$.

(5 marks)

QUESTION TWO

a) Determine the equation of a plane parallel to the plane $4x - 2y + z - 1 = 0$ and containing the point (2,6,-1).

(7 marks)

b) Using Charpits method, solve $32p^2z^2 + 18q^2z^2 + 8z^2 = 8$ and show that this solution is a family of ellipsoid with center on the x-y plane.

(13 marks)

QUESTION THREE

a) State and prove the Lagrange theorem for solving linear P.d.e. (10 marks)

b) Find a complete integral of the P.D.E. $x(p^2+q^2)=pz$ and deduce the solution which passes through the curves $x=a$ and $z^2=4y$. (10 marks)

QUESTION FOUR

a) Using Jacobi method, solve $2p_1x_1x_3+3p_2x_3^2+p_2^2p_3=0$ (5 marks)

b) Form a differential equation from $\Phi(x+y+z,x^2+y^2-z^2)=0$ (5 marks)

c) Find a solution of $xp-yq=z$ such that $p=3q$ when $x=2$ (10 marks)

QUESTION FIVE

a) Apply Lagrange theorem to solve

$$(y^3x-2x^4)p+(2y^4-x^3y)q=9z(x^3-y^3)$$ (14 marks)

b) Show that $T=F(y-3x)$ where T is an arbitrary differential function is a general solution of the equation $\frac{\partial T}{\partial x}+3\frac{\partial T}{\partial y}=0$ hence find the particular solution which satisfies the boundaries $T(0,y)=4\sin y$ (6 marks)

THE END