MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FOURTH YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 409: DIFFERENTIAL GEOMETRY

Date: 8th December, 2016 Time: 3.30 - 6.30 pm

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.
- Show all the necessary workings
- Start each question on a new page

QUESTION ONE (COMPULSORY) (30mks)

(a) Differentiate between simple closed curve and simple open curve. [2mks]

(b) If $\mathbf{u} = (3t^2+1)e_1 + \sin te_2$ and $\mathbf{v} = \cos te_1 + e^t e_3$, find

(i) $\frac{d}{dt}(\mathbf{u}\cdot\mathbf{v})$ [3mks]

(ii) $\frac{d}{dt}\mid \mathbf{v} \mid$ [3mks]

(c) (i) Determine the angle between the vectors $\vec{a} = e_1 - \frac{e_2}{3} - \frac{2e_3}{3}$ and $\vec{b} = \frac{e_1}{3} - \frac{2e_2}{3} - e_3$ [3mks]

(ii) Compute the length of the arc

$$\mathbf{x} = e^t \cos te_1 + e^t \sin te_2 + e^t e_3, \quad 0 \le t \le \pi$$

[4mks]

(iii) Find the unit tangent vector along the helix

$$\underline{X} = a\cos te_1 + a\sin te_2 + bte_3$$

where a and b are non-zero. [4mks]

(d) Find the values of λ for which the vector $\vec{A} = (\cos\lambda x, \sin\lambda x, 0)$ satisfy the differential equation $\frac{d^2A}{dx^2} = -9A$ [3mks]

(e) State the fundamental theorem of existence and uniqueness. [2mks]

(f) Show that the second fundamental form on the Monge patch $\vec{x} = ue_1 + ve_2 + f(uv)e_3$ is

$$II = (f_u^2 + f_v^2 + 1)^{-\frac{1}{2}}\left[f_{uu}du^2 + 2f_{uv}dudv + f_{vv}dv^2\right]$$

[6mks]

QUESTION TWO (20mks)

(a) Show that the tangent vectors along the curve $\vec{X} = ate_1 + bt^2e_2 + t^3e_3$, where $2b^2 = 3a$ make a constant angle with the vector $\vec{v} = e_1 + e_3$ [4mks]

(b) (i) Determine the point at which the curves

$$\vec{r_1(t)} = e^t i + 2\sin\left[t + \frac{\pi}{2}\right] j + (t^2 - 2)k$$

and

$$\vec{r_2(u)} = ui + 2j + (u^2 - 3)k$$

intersect and find the angle of intersection. [6mks]

(ii) Find the intersection of the x_1x_2 plane and the tangent line to the helix $\mathbf{x} = \cos te_1 + \sin te_2 + te_3$, $(t > 0)$. [4mks]

(c) Find the curvature along the curve $\mathbf{X} = (1 - \sin t)e_1 + (1 - \cos t)e_2 + e_3$ and hence determine the radius of curvature. [6mks]

QUESTION THREE (20mks)

(a) Consider the helix $X(t) = (a\cos t)e_1 + (a\sin t)e_2 + bte_3$ where $a > 0$ and $b \neq 0$, show that the curvature is $k(s) = \frac{a}{a^2+b^2}$ and torsion $\tau(s) = \frac{b}{\sqrt{(a^2+b^2)}}$ [15mks]

(b) Show that the first fundamental form on the surface of revolution

$$\vec{X} = (f(t)\cos\theta)\, e_1 + (f(t)\sin\theta)\, e_2 + g(t)e_3$$

is

$$I = f^2(t)d\theta^2 + \left((f'(t))^2 + (g'(t))^2\right) dt^2$$

[5mks]

QUESTION FOUR (20mks)

(a) Show that the unit tangent vector to the curve

$$r(t) = (4\cos t, \cos 2t, 2t + \sin 2t)$$

is $\mathbf{T} = (-\sin t, -\sin t \cos t, cos^2 t)$ and that the curvature is $\frac{1}{4}(1+\cos^2 t)^{\frac{1}{2}}$ [8mks]

(b) Find the length of the arc $u = e^{\frac{\theta \cot \beta}{\sqrt{2}}}$, $\theta = \theta$, $0 \le \theta \le \pi$, $\beta =$ constant on the cone $\mathbf{x} = (u\cos\theta)e_1 + (u\sin\theta)e_2 + ue_3$. [7mks]

(c) Find the surface area of the surface

$$\vec{X} = (u\cos v)e_1 + (u\sin v)e_2 + ku^2 e_3; \qquad o \le u \le 1,\ o \le v \le 1$$

[5mks]

QUESTION FIVE (20mks)

(a) Find the length of a curve $x(\vec{t}) = t^3 i + t^2 j$ from $t = 0$ to $t = 1$ and hence compare it to the straight line distance between the end points of the curve. [7mks]

(b) Show that the Gaussian curvature k on $\vec{X} = [(u+v), (u-v), uv]$ at $u = 1$, $v = 1$ is $-\frac{1}{16}$ [13mks]